2020 American Control Conference
Denver, CO, USA, July 1-3, 2020

On-board Supercapacitors Cooperative Charging Algorithm: Stability Analysis and Weight Optimization

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Abstract— **Onboard supercapacitors (SCs) charging usually requires fast charging with high current. In the constant current charging mode, a distributed multi-module charging system model is established to decompose the ultra high power from fast charging process. However, current overshoot and current imbalance in fast charging mode are prone to damage the reliability of charging system. In this paper, a Weight-optimizing Cooperative Charging Algorithm (WCCA) for onboard SCs is proposed to solve these problems. The key idea of WCCA is to design the dynamic correction weight factors to optimize the performance of current overshoot, imbalance and charging speed in three stages. Then, we prove the stability of the proposed algorithm through Lyapunov stability theorem. The impacts of current overshoot and imbalance on the charging system are evaluated through deviation distribution under noise mathematically. Compared with the existing cooperative charging strategies, WCCA scheme suppresses current imbalance and decreases current overshoot noticeably, while catching up with desired current quickly. Simulation and experiment results are provided to illustrate the feasibility and effectiveness of the algorithm.**

I. INTRODUCTION

As new energy storage devices, supercapacitors (SCs) have attracted considerable attention on charging energy storage electric vehicles (ESEVs), due to its superiorities of fast charging speed, long service life, high power density, no pollution, etc, [1]–[3]. Onboard SCs usually need to be charged in a short time at the docking station of ESEVs. It thus, requires a fast charging mode with a large current. The fast charging challenges constitute the one bottleneck of ESEVs application. Therefore, onboard SCs charging control is a thriving area of research in the field of ESEVs.

Numerous charging methods have been introduced in the literature with various objectives, e.g., enhancing charging performance, reducing energy loss and maximizing cycle life [4]–[7]. The methods for SCs charging control can be designed from the aspects of voltage balance, temperature monitoring, state evaluation, etc, [8]. For example, a temperature-suppression charging strategy is proposed to maximize lifetime of SCs [9]. PI observer is designed to estimate the internal capacitance voltage of an SC precisely to avoid the effect of equal series resistance [10].

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The choice of charging control models is critical to the design of the charging method. The charging control models are classified as centralized, decentralized and distributed. Among these models, distributed control is more promising due to the high robust and scalability. Thus, it has sparked considerable research interest in designing a reliable charging algorithm to guarantee charging performance [11], [12]. For instance, [13] proposes a distributed cooperative optimal control of DC microgrids to guarantee fairness of load sharing. In [14], the distributed cooperative control with novel topologies is applied for multiple DC electric springs to improve the power quality and voltage stability. When charging onboard SCs under a distributed control framework, current overshoot and current imbalance are unavoidable problems existing in the charging process, which damage the reliability and stability of charging system. Therefore, how to reduce current overshoot and suppress current imbalance is a vital issue to be solved.

The existing distributed cooperative charging methods can solve these problems to some extent [15]–[17]. However, there are still two major issues remained. On the one hand, the cooperative methods are not sufficient to solve them in terms of performance improvement. The cooperative weights are constant and cannot react to the current change in time while system fluctuates. On the other hand, stability theory of the existing cooperative methods for onboard SCs lacks of generalized proof [16]–[18], e.g., the stability of charging system consisted of three charging modules is proved [18]. Therefore, it is quite meaningful to give a general proof.

Motivated by above observation, we propose a weight-optimizing cooperative charging algorithm (WCCA) to improve and protect the charging performance with respect to overshoot, imbalance and charging speed. The main contributions are summarized as follows:

- We propose a WCCA charging scheme where dynamic correction weight factors are designed to improve and protect charging performance in current overshoot, imbalance and charging speed.
- We prove the stability of distributed cooperative charging system for onboard SCs through Lyapunov stability theory. The proof is also suitable for other algorithms.
- Compared with the existing distributed cooperative charging algorithm, current overshoot and imbalance are characterized to analyze charging performance mathematically. Simulation and experiment results are provided to validate the effectiveness of WCCA, which

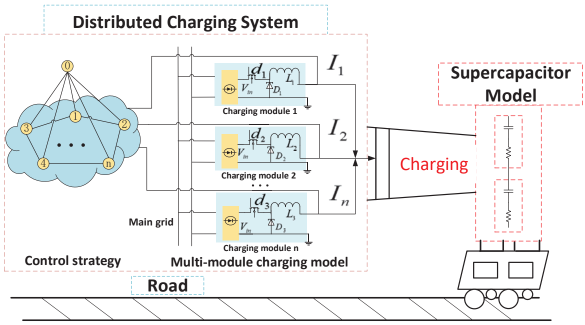

Fig. 1. Distributed multi-module charging system

show that WCCA makes current imbalance drop from 1.74% to 0.29% and overshoot decrease from 3.27% to 0.45%.

The rest of this paper is organized as follows. Section II introduces the distributed multi-module charging system model briefly. In Section III, the WCCA algorithm is proposed and theoretical analysis is given. Section IV are the simulation and experiment results. Finally, we summarize our work in Section V.

II. PROBLEM FORMULATION

In this section, a brief introduction to the basic graph theory is first given to describe communication links among charging modules. Then, we construct the distributed multi-module charging system model. Based on these, our research problems are formulated.

A. Graph Theory Basics

Suppose that there are n charging modules, denoted by $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$. $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is used to model the communication topology among modules, where $\mathcal{E}$ is the set of edges that connects two nodes. $(v_i, v_j) \in \mathcal{E}$ illustrates that node j can receive information from node i. $N_i = \{v_j : (v_i, v_j) \in \mathcal{E}\}$ denotes all neighbors of node i. The connection relationship for all modules is represented as the adjacency matrix $\mathcal{A} = [a_{ij}]_{n\times n}$, with $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. Degree matrix $\mathcal{D}$ is a diagonal matrix that is defined as $\mathcal{D} = \mathrm{diag}\{d_{ii}\}_{i=1}^{n}$ with $d_{ii} = \sum_{j\in N_i} a_{ij}$. The Laplacian matrix $\mathcal{L}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, which is symmetric for undirected graphs.

B. Charging System Model

The multi-module charging system for onboard SCs is shown in Fig.1. Each charging module contains a rectifier T_i and a buck regulator. The rectifier is used to convert AC power to DC input. Buck regulator consists of a MOSFET switch S_i, a diode D_i and a inductor L_i, where DC is regulated to charge SCs with a pulse width modulation mechanism. Each charging module is modeled by following linear time-invariant dynamic equation

$$L_i \frac{dI_i}{dt} = V_{in} d_i - U_c, \tag{1}$$

where V_{in} is the DC input voltage for each buck regulator and d_i is duty cycle adjusting the switch on-off time. U_c is the voltage across the SCs. I_i denotes the output current of charging module i.

For SCs, we exploit equivalent circuit model to describe their internal dynamic behaviors. The equivalent circuit model is connected in series by a resistor and a capacitor, which is not difficult to analyze [18], [19]. The physical model is modeled mathematically by

$$C_{eq} \frac{dU_c}{dt} = R_{eq} C_{eq} \frac{dI_c}{dt} + I_c, \tag{2}$$

where $I_c = \sum_{i=1}^{n} I_i$ charges the SCs. R_{eq} and C_{eq} are equivalent series resistance and capacitor, respectively.

C. Problem Description

There are three performance indexes of current overshoot, current imbalance and charging speed in constant current charging process. The reference current I_0 is defined in a warm-start mechanism [18], which satisfies

$$I_0(t) = \begin{cases} \frac{\tilde{I}}{t_0^2}\left[-(t-t_0)^2 + t_0^2\right], & if\ t \le t_0 \\ \tilde{I}, & if\ t > t_0 \end{cases} \tag{3}$$

where t_0 is the changing time of reference current from 0 to the desired current $\tilde{I}$ with $\tilde{I} = \frac{I_C}{n}$. Current overshoot exists in the process that output currents reach the desired current. Due to different kinds of uncertainty, e.g., noises, disturbances, the currents among modules cannot be totally the same. Thus, imbalance $\triangle I_i^j$ always exists, denoted by

$$\triangle I_i^j = \{I_j(t+1) - I_i(t+1) | I_i(t) = I_j(t)\}. \tag{4}$$

Current imbalance describes the deviation among charging modules in the steady state of charging system. Charging speed is characterized by the settling time when charging system enters the stable state.

In this paper, we mainly focus on these three interesting problems: i) how to design an algorithm to ensure the performance of charging system on these three indexes. ii) how to give a generalized proof to prove the stability of distributed charging system for onboard SCs. iii) how to quantify the performance indexes from a theoretical analysis perspective.

Remark 1: The reference current is no longer a constant value, but a parabola rises rapidly to the desired current. It is beneficial to reduce the excessive overshoot during constant current charging.

III. WCCA CHARGING SCHEME

A. WCCA Scheme Design

The existing distributed cooperative charging (DCC) strategy [15]–[17] is described as

$$\xi_i = g_{0i}(I_0 - I_i) + \sum_{j=1}^{n} a_{ij}(I_j - I_i).$$

ξ_i is current deviation, which consists of two parts. One is tracking the reference current. The other is balancing output currents among n charging modules. In the first term, g_{0i} is

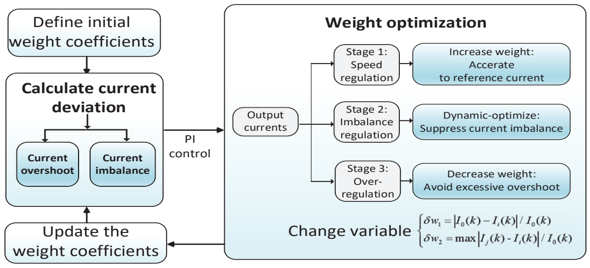

Fig. 2. Overview of WCCA scheme

a gain coefficient where $g_{0i} = 1$ if the leader node (reference current) pins to node i (charging module i), else $g_{0i} = 0$ [20]. In the second term, the gain coefficient is the element of adjacency matrix

Note that both of the gain coefficients are constant. The fixed weight coefficients cannot regulate output currents so flexibly that it leads to excessive overshoot and high current imbalance while system fluctuates. For example, due to the failure of one charging module, output currents of the remaining modules cannot be redistributed in time.

Thus, in this paper, WCCA is proposed, where the dynamic correction weight factors are designed to optimize the performance of charging system. The coordination is changed to

$$\xi_i = \lambda_i g_{0i} \left(I_0 - I_i\right) + \gamma_i \sum_{j=1}^{n} a_{ij} \left(I_j - I_i\right), \tag{5}$$

where λ_i and γ_i are the correction weight factors. λ_i optimizes the relationship between reference current I_0 and self module output current I_i. γ_i optimizes inter-communication among n charging modules. After current deviation is amplified by proportional integrator, the duty cycle d_i satisfies

$$d_i = k_p \left(T_i \xi_i(t) + \int_0^t \xi_i(\tau) d\tau \right), \tag{6}$$

where k_p is the integral coefficient of proportional integrator, T_i is the integral time constant.

Our goal is to minimize current imbalance and overshoot as small as possible simultaneously. The weight factors are optimized dynamically in conjunction with current state to shorten charging time when the output currents reach the reference current. Then, the overview of designing WCCA is illustrated in Fig. 2.

Definition 1: The stable fluctuation range is ε_u, in which the charging system is in steady state, i.e.,

$$|I_i(t) - \tilde{I}| < \varepsilon_u \tilde{I}, \tag{7}$$

where ε_u is constant.

The value of ε_u is selected according to classical control theory. In the stable range, system is stable, suppressing current imbalance is the key to optimization. Below the stable range, reaching the reference current quickly is the focus of optimization. Over the stable range, we should take the overshoot into account.

Therefore, the correction weight factors have different basis according to different current stages. The weight optimization rules are summarized into the following points.

- **Stage 1:** Below the stable range with $I_i(t) < (1 - \varepsilon_u)\tilde{I}$, I_i is in the state of speed regulation. It is noted that making output currents I_i catch up with the desired current $\tilde{I}$ is the optimization focus. We should increase λ_i to narrow the gap between $\tilde{I}$ and I_i quickly. γ_i remains the initial value to keep the balance among charging modules simultaneously.
- **Stage 2:** In the stable range with $|I_i(t) - \tilde{I}| < \varepsilon_u \tilde{I}$, I_i enters unbalanced stage, where output currents are close to reference current and current imbalance is the key to system optimization. We play more emphasis on reducing current imbalance among charging modules through changing weight value λ_i.
- **Stage 3:** Over the stable range with $I_i(t) > (1 + \varepsilon_u)\tilde{I}$, I_i is in over-regulated stage. Output currents exceed the stable range ε_u. λ_i needs to be decreased in time to avoid excessive overshoot.

The WCCA is described in Algorithm 1.

Algorithm 1 Weight Optimization Strategy

Input: λ, γ, I_i, I_j, I_0, t.
Output: weight λ, γ.

```
Initialize:λ, γ, I_i(1), I_j(1), I_0, t.
for k ← 1 to length(t) do
    Calculate the initial deviation ξ_i(k) using (5)
    Find the duty cycle d_i(k) using (6)
    Adjust I_i(k), I_j(k) according to the duty cycle
    if I_i(k) is in stage 1 then
        λ_i(k) ← λ_i(k) + δw_1,
        δw_1 = |I_0(k) − I_i(k)|/I_0(k).
    end if
    if I_i(k) is in stage 3 then
        λ_i(k) ← λ_i(k) − δw_1.
    end if
    if I_i(t) is in stage 2 then
        Find maximum and minimum current I_j(k)
        if I_j(k) − I_0 > 0 then
            λ_ij_max ← λ_ij_max − δw_2,
            δw_2 = max |I_j(k) − I_i(k)|/I_0(k).
        else
            λ_ij_min ← λ_ij_min + δw_2.
        end if
    end if
end for
```

B. System Stability Analysis

First, we construct a state space model. The first n state variables are output currents of n charging modules and the $n+1$ dimension state variable is voltage across the SCs. The duty cycle is used to denote the input variables. Based on (1) and (2), the state space model is written as

$$\dot{x} = Ax + Bu, \tag{8}$$

where $x = [x_1, \cdots, x_n, x_{n+1}]^T$ is the column vector of $n+1$ dimensions, $u = [u_1, u_2, \cdots, u_n, 0]^T$, and

$$A = \begin{bmatrix} 0_{n\times n} & -\frac{1}{L_{i,i=1,2,\cdots,n}} \\ \frac{1}{C_{eq}} & -R_{eq}\sum\limits_{i=1}^{n}\frac{1}{L_i} \end{bmatrix}_{(n+1)\times(n+1)},$$

$$B=\left[\begin{array}{cc}\operatorname{diag}\left(\frac{V_{in}}{L_{i,i=1,2,\cdots,n}}\right)_n & 0_{n\times 1}\\ \frac{R_{eq}V_{in}}{L_{i,i=1,2,\cdots,n}} & C\end{array}\right]_{(n+1)\times(n+1)}.$$

Assumption 1: n charging modules use the same inductance, i.e., we have $L_1 = L_2 = \cdots = L_n = L$.

Theorem 1: Assumption 1 holds and suppose that

$$\sum_{i=1}^{n} x_i \leq nI_0, \tag{9}$$

using Algorithm 1, the output currents of charging modules will converge to the reference current asymptotically, i.e., $\lim_{t\to+\infty} |I_i(t) - I_0| = 0,\ \forall i = 1, 2, \cdots, n$.

Proof: We prove the stability of charging system through Lyapunov stability theory. First, the Lyapunov function is given as $V(x) = \frac{1}{2}\sum\limits_{i=1}^{n}(x_i - I_0)^2$. From (8), the deviation of $V(x)$ satisfies

$$\dot{V}(x) = \sum_{i=1}^{n}\left[(x_i - I_0)\cdot\left(\frac{V_{in}u_i}{L_i} - \frac{x_{n+1}}{L_i}\right)\right]. \tag{10}$$

Under Assumption 1, we have

$$\begin{aligned}\dot{V}(x) =& \frac{V_{in}}{L}\sum_{i=1}^{n} x_i u_i + \frac{nI_0}{L}x_{n+1}\\ &- \frac{1}{L}\left(\sum_{i=1}^{n} x_i\right)x_{n+1} - \frac{V_{in}I_0}{L}\sum_{i=1}^{n} u_i.\end{aligned} \tag{11}$$

Since the charging mode is constant current, the charging time t satisfies $t = C_{eq}\left(U_c(t) - U_0\right)/I_c$, where U_0 is the initial voltage of SCs. Let U_0 be 0 without loss of generality, we have

$$x_{n+1}(t) = U_c(t) = \left(\sum_{i=1}^{n} x_i\right)t/C_{eq}. \tag{12}$$

Substituting (12) into (11), due to the duty cycle $d_i \in [0, 1]$, which is taken as the input variable u_i. We have $u_i = d_i \leq 1$. $\dot{V}(x)$ is expressed as

$$\begin{aligned}\dot{V}(x) \leq& -\frac{t}{LC_{eq}}\left(\sum_{i=1}^{n} x_i\right)^2 + \frac{nI_0 t + V_{in}C_{eq}}{LC_{eq}}\sum_{i=1}^{n} x_i\\ &- \frac{V_{in}I_0}{L}\sum_{i=1}^{n} u_i.\end{aligned} \tag{13}$$

Next, judge the state of $\dot{V}(x)$. Make $X = \sum\limits_{i=1}^{n} x_i$, then, $\dot{V}(x) = M(X)$, $M(X) = JX^2 + PX + Q$, where

$$J = -\frac{t}{LC_{eq}}, P = \frac{nI_0 t + V_{in}C_{eq}}{LC_{eq}}, Q = -\frac{V_{in}I_0}{L}\sum_{i=1}^{n} u_i.$$

The curve of function $M(X)$ is a parabola with the opening down and the axis of symmetry in the positive semi-axis of x. X is the sum of output currents of charging modules. Note that the current value X is at least 0 (a negative current indicates that current direction is opposite to specified positive direction). When $X = 0$, $M(X) = Q \in [-\frac{nV_{in}I_0}{L}, 0] \leq 0$. When $X = -\frac{P}{2J}$, $M(X)$ has the maximum value $V_{max} \in [\frac{(nI_0 t - V_{in}C_{eq})^2}{4tLC_{eq}}, \frac{(nI_0 t + V_{in}C_{eq})^2}{4tLC_{eq}}] \geq 0$. The intersections of $M(X)$ and x axis are $X_1 = -\frac{P-\sqrt{P^2-4JQ}}{2J} = \frac{V_{in}C_{eq}}{t}$, $X_2 = -\frac{P+\sqrt{P^2-4JQ}}{2J} = nI_0$.

In the constant current charging mode, if $X = \sum\limits_{i=1}^{n} x_i \leq I_c$ is constant, through (12) we can get $X_2 < X_1$. For all $X \leq nI_0$, there always exists $M(X) \leq 0$.

Therefore, $\dot{V}(x) < 0$ is always established. So charging system is globally asymptotically stable. ■

Remark 2: The stability conditions (9) is suitable for other algorithms. Note that the existing literature only prove the stability of the charging system in the case of low dimensions. We extend it and the proof in this paper is equally applicable to any finite dimension.

C. Discussion

In this section, we discuss the impacts of current overshoot and imbalance on charging system mathematically. First, note that noise is unavoidable in the circuit [21]. Consider WCCA strategy (5) with noise θ_i, which is assumed to be a Gaussian white noise with an uncertain variance σ:

$$\mathbb{E}(\theta_i) = 0, \quad \mathbb{E}(\theta_i\theta_j) = \begin{cases}\sigma > 0 & \text{if } i = j\\ 0 & \text{if } i \neq j\end{cases}, \tag{14}$$

where $\mathbb{E}(\cdot)$ is expectation operator and Gaussian noise $\{\theta_i | i \in (1, \cdots, n)\}$ and $\{\theta_j | j \in N_i\}$ are added at $t \in (t_1, t_2)$ time when charging system is stable. The current deviation with noise is denoted by

$$\xi_i^\theta = \lambda_i g_{0i}\left(I_0 - I_i - \theta_i\right) + \gamma_i\sum_{j=1}^{n} a_{ij}\left(I_j - I_i + \theta_j - \theta_i\right).$$

Combined with (14), we have

$$\mathbb{E}(\xi_i^\theta) = \mathbb{E}(\xi_i), \quad D(\xi_i^\theta) = -D(\theta_i)\alpha^2 + D(\theta_j)\beta^2 + D(\xi_i),$$

where $\alpha = \lambda_i g_{0i} + \gamma_i\sum\limits_{j=1}^{n} a_{ij}$, $\beta = \gamma_i\sum\limits_{j=1}^{n} a_{ij}$. Related to cooperative charging strategy with Gaussian noise, we obtain current deviation attenuation ratio

$$\eta_i = \frac{\left\|\xi_i^\theta - \xi_i\right\|_2}{\left\|\zeta_i^\theta - \zeta_i\right\|_2} = \begin{cases}\lambda_i & i = j,\\ \frac{\xi_i(\theta_i)}{\zeta_i(\theta_i)} & i \neq j,\end{cases} \tag{15}$$

where ζ_i denotes current deviation of DCC strategy and ζ_i^θ is current deviation with noise. Owing to $\lambda_i, \gamma_i \in (0, 1)$ when system is stable, $0 < \eta_i < 1$ is obtained directly. Hence, it is demonstrated noticeably that WCCA is more beneficial for cutting down current deviation than DCC under the noise.

Current deviation is composed of overshoot and imbalance. The probability that, current deviation is around 0, is used to describe the size of overshoot and imbalance, which is calculated by $P\left(\left|\xi_i^\theta\right| < \varepsilon\right) = \int_{-\varepsilon}^{\varepsilon}\varphi\left(\xi_i^\theta\right)d\left(\xi_i^\theta\right)$. ε represents the confidence interval. The probability density distribution (PDF) of current deviation is shown in Fig.3, which shows that WCCA has a higher probability of zero deviation than DCC under the noise. Thus, it means that

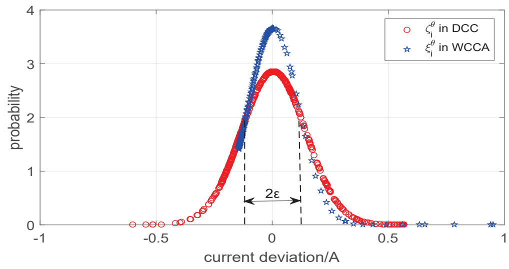

Fig. 3. Comparison of PDF between WCCA and DCC

current overshoot and imbalance are effectively suppressed under the control of WCCA.

IV. Performance Evaluation

In this section, we exploit simulation and experiment results to verify the feasibility and effectiveness of WCCA.

A. Simulation

Consider a three-module charging system. The parameters of charging system are set based on actual physical devices as follows: input voltage $V_{in} = 24V$, reference current $\tilde{I} = 5A$, equivalent capacitor $C_{eq} = 100F$, equivalent resistor $R_{eq} = 0.005\Omega$. The three inductances $L_1 = 10mH, L_2 = 9mH, L_3 = 8mH$. The proportional parameter $K_i = 1.5$, the integral parameter $T_i = 1/60s$. Stable fluctuation range $\varepsilon_u = 0.2$, which provides the highest precision. It is fully connected between the reference current and output currents of charging modules and among the modules. Then, Laplace matrix $\mathcal{L} = [2 - 1 - 1; -1\ 2 - 1; -1 - 1\ 2]$ and pinning matrix $G = [1\ 0\ 0; 0\ 1\ 0; 0\ 0\ 1]$.

The simulation results of DCC and WCCA are shown in Fig. 4 and Fig. 5, respectively. Comparing the two figures, it is easy to find that WCCA is stable in 0.12 seconds but DCC is not stable in the range of $0.08 - 0.15$ seconds. In addition, WCCA reduces current overshoot and current imbalance related to DCC. The specific simulation results are shown in Table I. We know that WCCA can speed up charging from original $0.1357s$ to $0.1008s$. Current imbalance drops from 1.74% to 0.29% and current overshoot decreases from 3.27% to 0.45%. In short, the charging performance is improved in charging speed, current overshoot and current imbalance under the control of WCCA.

TABLE I
COMPARISON OF SIMULATION RESULTS

Method	DCC	WCCA
Settling Time	$0.1357s$	$0.1008s$
Current imbalance	1.74%	0.29%
Current overshoot	3.27%	0.45%
Current steady state deviation	$0.0042A$	$0.0006A$

B. Experiment

1) Experiment Setup: The SC charging experiment platform is depicted in Fig. 6. It's mainly composed of three charging modules, 24V DC power supply, SCs and a development board of STM32F205RBT6. The communication links among charging modules are achieved through the control program on the development board. Each program controls the corresponding output currents. The charging system parameters are the same as the simulation, except for the identical inductance values $L = 10mH$.

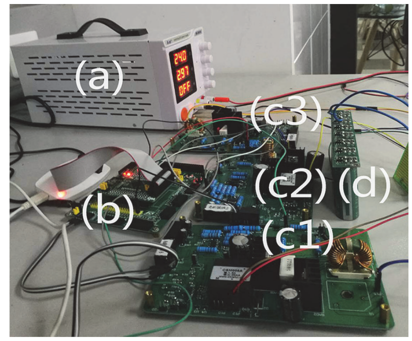

Fig. 6. Supercapacitor Charging Experiment Platform. (a) 24V DC power source. (b) Development board. (c1),(c2),(c3) Three charging modules. (d) Supercapacitors

2) Experiment Results: The experiment results are shown in Fig. 7. Since the sampling frequency is $100HZ$, dividing the number of abscissas by 100 is the time variable in seconds. Consider the effect of changing the weight coefficients on the performance of the charging system. The decentralized charging strategy (DCS) is equivalent to the case of $\{\lambda = 1, \gamma = 0\}$. Relative to the distributed cooperative charging strategy (DCC), which is in the case of $\{\lambda = 1, \gamma = 1\}$, DCS only considers the difference between the output currents and the reference current and has a large current overshoot. WCCA is designed through changing correction weight factors to improve the charging performance. Compared with DCC, WCCA can balance output currents among charging modules and decrease overshoot effectively. Therefore, the effectiveness of WCCA is validated.

V. Conclusions

In this paper, we focus on how to improve and protect the performance of charging system in terms of current overshoot, imbalance and charging speed. We propose a weight-optimizing cooperative charging algorithm (WCCA) to solve the problems. The key idea of WCCA is to design the dynamic correction weight factors to optimize these performance indexes. Then, we give a general stability proof of distributed cooperative charging system for onboard SCs. The impacts of current overshoot and imbalance on system performance are analyzed mathematically. Finally, the simulation and experiment results are demonstrated to illustrate the feasibility and effectiveness of WCCA.

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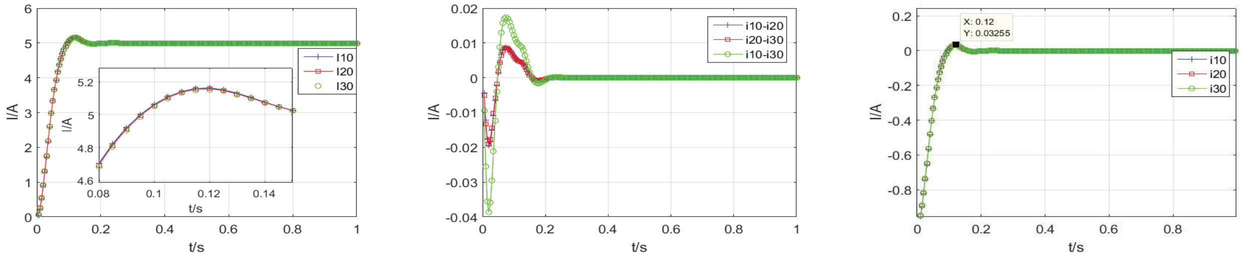
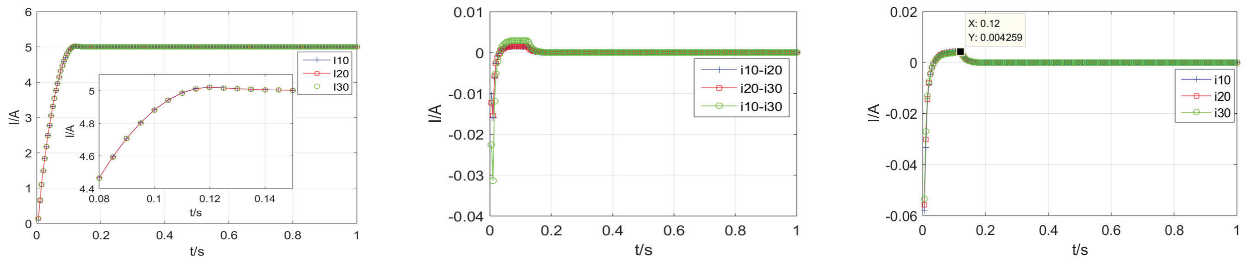


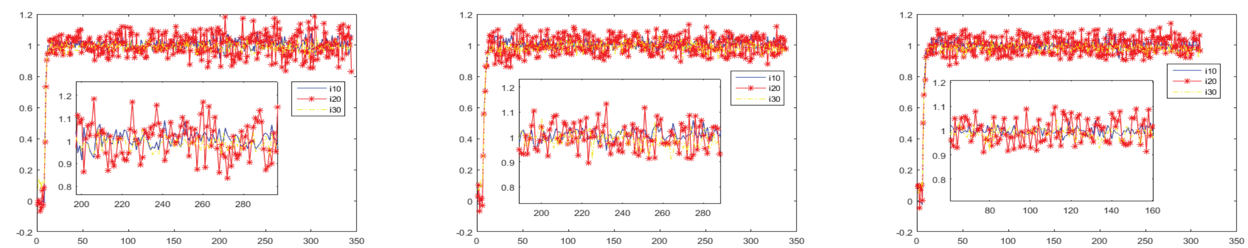
(a) Output currents of three charging modules (b) Current imbalance among charging modules (c) Current overshoot among charging modules

Fig. 4. Distributed Cooperative Charging strategy



(a) Output currents of three charging modules (b) Current imbalance among charging modules (c) Current overshoot among charging modules

Fig. 5. Weight-optimizing Cooperative Charging Algorithm (WCCA)



(a) Decentralized Charging Strategy (b) Distributed Cooperative Charging (c) WCCA Charging Algorithm

Fig. 7. The output currents comparison of DCS, DCC and WCCA charging strategy

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