Optimal Topology Recovery Scheme for Multi-robot Formation Control

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***Abstract*—In applications of formation control, maintaining the original topology is of vital importance, e.g., military surveillance. How to recover the original topology based on the robotic mobility is currently an open issue. In this paper, therefore, a distance-based topology recovery scheme for multi-robot formation control is proposed to tackle this problem. First, we derive the novel motion model of a robot, where the model parameters can be estimated based on the classic least square method (LSM). Thus, the historical data is exploited to estimate a leader's motion, so that its follower can still track it even when the link fails. Then, we provide theoretical guarantees for successful topology recovery. Both minimum-time and minimum-distance controllers are designed for the follower to reconnect with its leader, given different velocity constraints and different relative distance and bearing of the two robots. Furthermore, the successful recovery probability is obtained when the proposed velocity and relative position conditions cannot be satisfied. Simulations illustrate the effectiveness of the proposed topology recovery scheme.**

I. Introduction

Multi-robot system has received considerable attention in the last decades. It relates to a broad range of applications, such as exploration of unknown environments, military surveillance and reconnaissance, search and rescue in disaster sites [1]. In those applications, the robots usually work as a formation and need to communicate with each other to ensure effective cooperation. Thus, stable connectivity/topology is crucial for the formation control to accomplish the cooperative task. However, it is almost inevitable that the system will suffer communication failures due to hazardous factors in the environment. It thus is necessary to design efficient techniques to deal with the connectivity problem. Since the communication network among the robots is generally modeled as a graph, ensuring the connectivity of the formation can be mathematically translated into keeping the graph is connected.

Many efforts have been devoted to investigating the problem of connectivity maintenance for formation control. Those work can be divided into two types, maintain local connectivity [2]–[6] or global connectivity [7]–[11]. The former kind requires the communication links active all the time if they are preset to be connected, which is too difficult to implement at times. The latter kind can be seen as the former one's relaxation, which is mainly based on algebraic connectivity (i.e., the value of the second-smallest eigenvalue of the Laplacian matrix of the graph). The main idea is to keep the eigenvalue strictly greater than zero to guarantee connectivity.

However, few studies have investigated how to recover the connectivity when inevitable connectivity failures occur [12]. In the field of wireless sensor and actor networks (WSANs), the authors in [13] propose distributed actor recovery algorithm (DARA) to replace a dead actor by a healthy one. Furthermore, [14] considers avoiding obstacles while relocating sensors. The key idea of this kind technique is the deployment of backup nodes, which is not practical for mobile robots network. There are also literature aiming at consensus recovery for cut-node or cut-link failures in multi-robot system [15], [16]. Nevertheless, the original network topology is changed using these methods. Moreover, most research on connectivity problem are based on undirected graphs, which is more simple than directed graph cases.

It still remains an open and challenging problem for a mobile robot, who loses communication with its neighbor, to restore the lost link and recover the original directed topology. Therefore, in this paper, we propose a connectivity recovery scheme for directed multi-robot network, where the structure of the topology needs to be exactly the same as the one before connectivity failure. The main work includes three aspects. First, when a robot loses its communication link with another one and cannot receive the information, it utilizes stored historical data to estimate its neighbor's motion. Second, based on the estimation, different control strategies are proposed for different velocity constraints and relative positions. Third, we obtain the guarantees for successful connectivity recovery and the success probability when the guarantees do not hold.

Our work to recover the topology of original directed graph is meaningful and significant due to the following reasons: i) Modeling the communication architecture as a directed graph, which weakens the premise of most existing work, agrees better with practicality, because the communication pattern is commonly asymmetric [17]; ii) Compared with methods that allow topology change, recovering the original topology can bring less cost, and doesn't need to alter the internal cooperation and protocols of the robots. Also, every single robot has only limited information of its fixed neighbors, permitting a degree of the privacy/security for the whole system [18], e,g, a compromised robot will leak the system information it hold [19].

The remainder of this paper is organized as follows. In Section II, the system dynamic model of consensus-based formation control is introduced. In Section III, the motion

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estimation and control strategy are proposed, as well as the effectiveness analysis of the method. Simulation results are shown in Section IV. Finally, Section V concludes this paper.

II. Problem Formulation

A. Graph Theory Basics

Let $\mathcal{G}$=($\mathcal{V}$,$\mathcal{E}$) be a directed graph that models the communication topology among agents, where $\mathcal{V} = \{v_1, \cdots, v_N\}$ is the finite set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge $(v_j, v_i) \in \mathcal{E}$ indicates that v_i can receive information from v_j. The adjacency matrix $A = [a_{ij}]_{N\times N}$ of a graph is defined such that $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise.

A directed path is a sequence of nodes v_1-v_2-$\cdots$-v_r such that $(v_i, v_{i+1}) \in \mathcal{E}$, $i \in \{1, 2, \cdots, r-1\}$. A directed graph has a (directed) spanning tree if there exists at least one node having a directed path to all other nodes. Let $N_i = \{j \in \mathcal{V} : a_{ij} \neq 0\}$ be the set of neighbors of node v_i, and $L = \Delta - A$ be the Laplacian matrix of the graph $\mathcal{G}$, where $\Delta = \text{diag}(A \cdot \mathbf{1})$. A directed graph is said to have a loop, if there exits a path with the start node and the end node being the same. $\mathcal{G}$ must have a spanning tree to guarantee the consensus.

Let $L = MJM^{-1}$ be the Jordan decomposition of L, where $M = [\ z_1 \quad z_2 \quad \cdots \quad z_N\ ]$ is the transformation matrix, $M^{-1} = [\ w_1 \quad w_2 \quad \cdots \quad w_N\ ]^T$, and $J = \text{diag}(J_i)$ with J_i being the Jordan block of eigenvalue λ_i. For ease of notation and discussion, supposing all eigenvalues of L are distinct. The eigenvalues are ordered as $|\lambda_1| \leq |\lambda_2| \leq \cdots \leq |\lambda_N|$, and

$$(\lambda_i I - L)z_i = 0, \ w_i^T(\lambda_i I - L) = 0, \tag{1}$$

where left eigenvectors w_i are normalized, such that ${w_i}^T z_i = 1$. When graph $\mathcal{G}$ has a spanning tree, L has rank $N-1$, i.e., $\lambda_1 = 0$ and its right eigenvector is any constant vector $c\mathbf{1}$, and $|\lambda_2| > 0$ holds [20].

B. System Dynamic Model

Let $p = [p_1^T, p_2^T, ..., p_N^T]^T$ be the position vector of the multi-robot system, and $p_i \in \mathbb{R}^M$ represents position of robot i in global coordination. Denote $u = [u_1^T, u_2^T, ..., u_N^T]^T$ as the control input vector, where $u_i \in \mathbb{R}^M$. By refering to [21] and taking the formation shape into consideration, the first order dynamics of each robot is given by

$$\dot{p}_i = u_i = \beta \sum_{j\in N_i} a_{ij}(p_j - p_i - r_{ij}), \tag{2}$$

where β is a positive constant, r_{ij} is the predefined relative distance vector between robot i and robot j. Using the well-known *R*-disk communication model [22], a_{ij} is defined by

$$a_{ij} = \begin{cases} 0, \ \| p_i - p_j \| > d_j, \\ 1, \ \| p_i - p_j \| \leq d_j, \end{cases} \tag{3}$$

where d_j is the communication radius of robot j. The robots may be heterogenous due to their different communication modules and sensors, i.e., we may have $d_i \neq d_j$. With (2), we have the global dynamic model as

$$\dot{p} = u = -\beta L \otimes p - h, \tag{4}$$

where $\otimes$ is Kronecker product, and h satisfies

$$h = \beta \Big[\sum_{j\in N_1} a_{1j} r_{1j}^T \quad \sum_{j\in N_2} a_{2j} r_{2j}^T \quad \cdots \quad \sum_{j\in N_N} a_{Nj} r_{Nj}^T \ \Big]^T.$$

By referring to [1], the preset formation shape will be formed using (4), and we also say consensus is reached, i.e.,

$$\hat{e}_i = \big[p_i - p_1 - r_{1i} \quad \cdots \quad p_i - p_N - r_{Ni}\big]_N^T = \mathbf{0}. \tag{5}$$

For simplicity, we assume $p_i \in R^1$ in the reminder of this paper. All the results generalize without difficulty to robot's motion in 2-D or 3-D space.

C. Problem of Interest

Consider the formation is leader-follower architecture and there is a unique root leader. While robots are moving in an unknown environment, some links are likely to break for many reasons. For example, part of the formation can encounter obstacles sometimes. To avoid these obstacles, the robots will take corresponding action, deviating from its desired trajectory. During the adjustment, the communication link has a large chance to fail, because the deviation is likely to make the relative distance between a robot and its neighbors exceed the communication range. After the link of a pair of robots is lost, how the follower R_i recovers the link with its leader R_j based on the mobility is considered in this paper. The challenges mainly lie in two parts: i) how to estimate R_j's motion accurately so that the recovery is possible; ii) how to design control strategy to guarantee the connectivity between the two robots will be recovered successfully and optimally.

III. Main Results

A. Preliminary Analysis

When a communication link fails, there are two situations for the resulting system topology: connected and disconnected. First, we briefly analyze the former situation.

Lemma 1. *If there are some edges broken but the system graph is still connected, consensus will still be reached.*

This lemma can be easily proved by transforming Laplacian matrix L into Jordan normal form ,referring to [21], [23]. As long as the broken topology is still connected, the lost link will always be recovered autonomously during the process.

However, for disconnected situations, some robots of the formation cannot receive information directly or undirectly from the root leader, thus the formation shape is unable to be formed merely based on control law (2).

Remark 1. *In our scenario, there is a special case when a failed link causes the graph disconnected: the lost link is part of a loop in the original topology. This situation indicates there is one more static leader in the formation, resulting in undesirable obstruction for the whole system. It is not within the scope of this paper.*

To recover the formation topology in disconnected situations, we first propose a motion estimation method utilizing historical data. Then, based on the estimation, control laws for

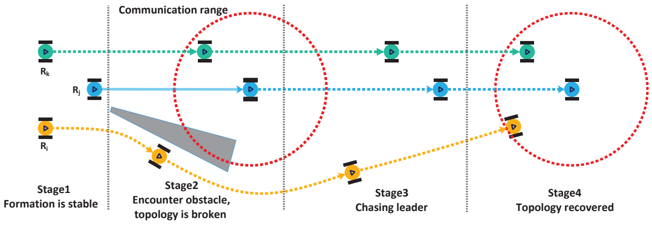


Fig. 1. The whole process of the scenario we focus on in this paper, which can be divided into 4 stages. R_j is the leader, R_i and R_k are R_j's two followers. When R_i loses communication with R_j, it will recover the topology based on the proposed motion estimation and control strategy.

the robot are designed for different situations. The following basic assumptions are made to formulate the problem.

Assumption 1. *Consider a pair of robots R_j and R_i, where R_i receives information from R_j (we call R_j is a leader of R_i) and can store the information with fixed memory. If the memory is full, the data only needs to be updated online.*

Assumption 2. *The root leader's movement is regular, i.e., it can be modeled by a function of time explicitly, which is continuous everywhere and indifferentiable in finite points.*

Fig. 1 shows an example of the whole process. First, the robots need to form the preset formation shape. Second, when the system topology is broken at time t_0, R_i estimates the velocity curve of R_j using received historical data. Third, obstacle-avoidance is achieved at time t_1, and R_i starts implementing the new control strategy based on the estimated $\tilde{p}_j(t)$. At last, R_i moves to an estimated point and reconnects with R_j at time t_2. In the following, we denote obstacle-avoidance time slot $\Delta T_{01} = t_1 - t_0$, chasing time slot as $\Delta T_{12} = t_2 - t_1$, $\Delta T_{02} = t_2 - t_0$ and d_c is the communication range of R_j.

B. Motion Estimation

To achieve a specific formation, the robots need to adjust their motion by the information they received from neighbors. By Assumption 2, we have obtained the following lemma.

Lemma 2. *For a robot R_j in the formation, its velocity function can be written as*

$$v_j = \sum_{i=1}^{N-1} a_i(t)e^{-b_i t} + c(t), \tag{6}$$

where b_i are constants, $a_i(t)$ and $c(t)$ are functions determined by the leader's velocity and system topology.

Proof. Considering the whole formation's motion in one direction, the global dynamics of n robots are given by

$$\dot{x}(t) = -Lx(t) + u_0(t), \tag{7}$$

where $u_0(t)$ is a n-dimension vector and all its elements are zero except the one representing the leader. Multiply e^{-Lt} on both sides of (7) and integrate form 0 to t, then we obtain

$$x(t) = e^{-Lt}x(0) + \int_0^t e^{-L(t-\tau)}u_0(\tau)d\tau. \tag{8}$$

Using modal decomposition, (8) is rewrited in the terms of Jordan form of L, given by

$$\begin{aligned} x(t) &= e^{-Lt}x_0 + \int_0^t e^{-L(t-\tau)}u_0(\tau)d\tau \\ &= \sum_{i=2}^{N}\left(z_i e^{-\lambda_i t}w_i^T x_0 + \int_0^t z_i e^{-\lambda_i(t-\tau)}w_i^T u_0(\tau)d\tau\right) \\ &\quad + z_1 w_1^T\left(x(0) + \int_0^t u_0(\tau)d\tau\right), \end{aligned} \tag{9}$$

where the first term is the sum of exponential function, and the second term is the integral of u_0.

Differentiating (9) with respect to t, the form of $\dot{x}(t)$ is similar except that the second part is a weighted average sum of u_0. With the representation of the weights simplified, each element of $x(t)$ is given by

$$\dot{x}_j(t) = v_j(t) = \sum_{i=1}^{N-1} a_i(t)e^{-b_i t} + c_j(t), \tag{10}$$

where b_i represents the eigenvalues, $a_i(t)$ and $c_j(t)$ are determined by the leader's velocity $u_0(t)$. □

Note that most general functions can be expressed as summation of polynomial function based on Taylor Expansion, and modeling $c(t)$ as a polynomial function is reasonable and practical as long as the order is high enough.

Remark 2. *Since $\lambda_1 = 0$, the attenuation speed is mainly determined by λ_2. Thus, we directly revise the sum term in* (10) *as a single exponential term. The errors brought by this simplification will be discussed later.*

For simplicity and without losing generality, we assume the formation leader moves with a uniform velocity. It follows that $a_i(t)$ and $c(t)$ are constant. Then, we have

$$v_j = \bar{a}_j e^{-b_j t} + c, \quad p_j = \int_{t_0}^t v_j dt + p_j(t_0). \tag{11}$$

However, in most applications, the information R_i received from R_j are usually inaccurate, for the self-motion information collected by its embedded sensors have certain bias, or network channels can be affected with some unavoidable noises. Taking this factor into consideration, we further revise (11) as

$$\tilde{v}_j = \bar{a}_j e^{-b_j t} + c + \varepsilon_j, \quad \tilde{p}_j = \int_{t_0}^t \tilde{v}_j dt + p_j(t_0), \tag{12}$$

where ε_j is bounded random noise.

Based on the model given by (12), we obtain the parameters by solving the following least square problem

$$\min_{\bar{a}_j, b_j, c} \sum_{i=1}^{k}\left(\bar{a}_j e^{-b_j t_i} + c - SD(t_i)\right)^2,$$

where SD is the stored data set of k moments. It should be noted that (12) is reasonable and sufficient to estimate R_j's motion due to the following,

1) In multi-robot formation control, tree graph is one of the most commonly used topology structure;
2) When consensus is reached, the exponential decay term of (12) is ignored. And when not, other eigenvalues's terms except λ_2 can be considered as part of ε.

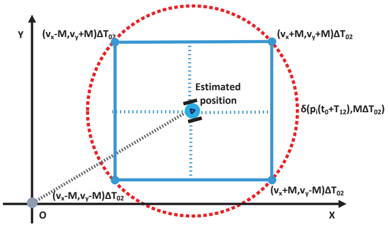


Fig. 2. After time slot T_{12}, R_j lies in the uncertainty area, i.e., the circle region $\delta\left(p_i(t_1+T_{12}), MT_{02}\right)$. 'O' is the estimated position of R_j when R_i finishes obstacle-avoidance at t_1.

C. Uncertainty Analysis

After obtaining the estimation of R_j, the main problem turns to how to guarantee the success for R_i to reconnect with R_j even with the existence of uncertainty ε.

It's natural and reasonable to assume the value of uncertainty ε distributes on both positive and negative domain with a bound. When the velocity curve of R_j is regressed, we are able to make a statistical analysis, computing the biggest error between the fitted function and historical data, given by

$$M = \max\{|SD(t_i) - FD(t_i)|, i = 1, 2, \cdots, k\}, \quad (13)$$

where FD is the fitted data set of k moments.

It should be noted that the bound M is a relative quantity between the estimated function and the noisy data, not the absolute bound of ε itself. Given M, we have the following two kinds estimation:

$$\tilde{v}_{j1} = a_j e^{-b_j t} + c_j - M, \quad \tilde{v}_{j2} = a_j e^{-b_j t} + c_j + M. \quad (14)$$

Obviously, $v_j \in [\tilde{v}_{j1}, \tilde{v}_{j2}]$.

Supposing R_i will meet with estimated R_j at t_2, we have

$$p_i(t_1) + \int_{t_1}^{t_2} v_i(t)dt = \tilde{p}_j(t_1) + \int_{t_1}^{t_2} \tilde{v}_j(t)dt. \quad (15)$$

(15) is a representation of absolute coordinates, for simplicity, we use representations based on relative distance and bearing hereafter. To catch up with R_j as fast as R_i can, the following condition must hold:

$$(v_i \Delta T_{12})^2 = \left(\int_{t_1}^{t_2} \tilde{v}_j(t)dt\right)^2 + {d_0}^2 - 2\left(\int_{t_1}^{t_2} \tilde{v}_j(t)dt\right) d_0 \cos\alpha.$$

If R_j has kept following the leader or the exponential term of $\tilde{v}_j$ is small and ignorable, the condition is expressed as

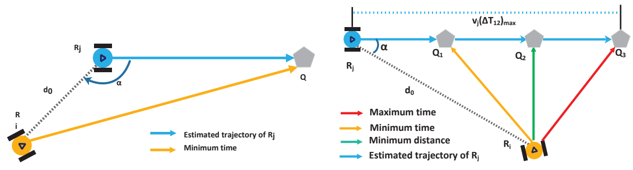
$$(v_i \Delta T_{12})^2 = (\tilde{v}_j \Delta T_{12})^2 + d_0^2 - 2 d_0 \tilde{v}_j \Delta T_{12} \cos\alpha. \quad (16)$$

Further, we have

$$v_i = f(\Delta T_{12}) = \left(\tilde{v}_j^2 + (d_0/\Delta T_{12})^2 - 2 d_0 \tilde{v}_j \cos\alpha / \Delta T_{12}\right)^{\frac{1}{2}}. \quad (17)$$

Figure 2 illustrates the influence of the uncertainty. Suppose a time period ΔT_{02} has past after the link lost at t_0, and R_j is at the point 'O'. The circle region represents the possible location where R_j really is, denoted as $\delta(\tilde{p}_j(t_0 + T_{02}), M\Delta T_{02})$. The longer the chasing time T_{12} is, the more uncertain P_j will be. The four vertex represents four kinds of worst cases.

Since the uncertainty will continuously grow, a natural question is what is the worst case we can tolerate and how to design corresponding control strategy.



(a) $\alpha > \alpha_0$: if $v_i^{max} \geq v_i^c$, R_i is able to catch up with R_j at estimated point Q and recover the link successfully.

(b) $\alpha \leq \alpha_0$: $D(\alpha, d_0)$ is relatively short for R_i, and the velocity constraints is loosen.

Fig. 3. Different situations for R_i to catch up with R_j.

TABLE I
NOTATION OF COMMONLY USED VARIABLES

Notation	Meaning
T_c	$T_c = d_c/(\sqrt{2}M)$, the maximum acceptable time.
$\Delta T_{12}^{\max}$	$\Delta T_{12}^{\max} = T_c - \Delta T_{01}$, the maximum chasing time.
v_i^c	The critical speed when $\Delta T_{12} = \Delta T_{12}^{max}$.
α_0	The critical bearing angle when $\Delta T_{12} = \Delta T_{12}^{max}$.
ΔT_{12}^*	$\Delta T_{12}^* = d_0/(\tilde{v}_j \cos\alpha)$ where $\alpha \in (0, \alpha_0)$.
v_i^*	The minimum feasible velocity when $\alpha \leq \alpha_0$.

D. Control Strategy and Recovery Probability

After R_i finishes obstacle-avoidance at t_1, there might be two situations in terms of the relative position between R_i and R_j, as shown in Fig. 3(a) and Fig. 3(b). Note that R_j's velocity is estimated by R_i and the relative distance d_0 and relative bearing α is also known for R_i.

For ease of discussion, definitions of some commonly used variables hereafter are presented in Table I. Then, we formulate different situations as the following conditions:

C1: $T_c > \Delta T_{01}$ and $v_i^{max} \geq \tilde{v}_j$.
C2: $\alpha > \alpha_0$ and $v_i^{max} \geq v_i^c$.
C3: $\alpha \leq \alpha_0$ and $v_i^{max} \geq \tilde{v}_j \tan\alpha$.

Next, we investigate how to design the optimal control strategy that guarantees the success of connectivity recovery given certain objectives mathematically. Taking minimum-time as the objective, we formulate the problem as:

$$\begin{aligned} \min_{v_i} \quad & \Delta T_{12}(v_i) \\ \text{s.t.} \quad & \text{C}_1,\ (16),\ \text{and } v_i \leq v_i^{max}, \\ & (v_i^c \leq v_i \text{ and C2}) \text{ or } (v_i^* \leq v_i \text{ and C3}). \end{aligned} \quad (18)$$

Taking minimum-distance as the objective, we formulate the problem as:

$$\begin{aligned} \min_{v_i} \quad & v_i \Delta T_{12}(v_i) \\ \text{s.t.} \quad & \text{C1},\ \text{C3},\ \text{and } (16), \\ & v_i^{max} \geq \tilde{v}_j \tan\alpha \text{ and } v_i^* \leq v_i \leq v_i^{max}. \end{aligned} \quad (19)$$

Then, we obtain the following theorems, which show the optimal control strategies corresponding to the above optimization objectives, respectively.

Theorem 1. *When C1 and C2 hold, for* $\forall v_i \in [v_i^c, v_i^{max}]$, *the link between* R_i *and* R_j *can be recovered successfully. And the optimal control strategy for the minimum chasing time is*

$$u_i^* = v_i^{max}, \quad (20)$$

from which the minimum chasing time is given by

$$\Delta T_{12}^{min} = \frac{-d_0 \tilde{v}_j \cos\alpha + d_0 \sqrt{\tilde{v}_j^2 \cos^2\alpha + (v_i^{max})^2 - \tilde{v}_j^2}}{(v_i^{max})^2 - \tilde{v}_j^2}. \quad (21)$$

Proof. By (17), we obtain the critical speed $v_i^c = f(\Delta T_{12}^{max})$. When $\alpha \geq \pi/2$, $\frac{dv_i}{d\Delta T_{12}} < 0$, thus (17) is monotonically decreasing. When $\alpha < \pi/2$, treat $\frac{1}{\Delta T_{12}}$ as the argument, then $f(\frac{1}{\Delta T_{12}})$ is a quadratic function of $\frac{1}{\Delta T_{12}}$, and v_i reaches the minimum value $v_i^* = \tilde{v}_j \sin\alpha$ when $\Delta T_{12} = \Delta T_{12}^* = d_0/(\tilde{v}_j \cos\alpha)$. Specially, when $\Delta T_{12}^{max} = \Delta T_{12}^*$, we denote the critical bearing angle as

$$\alpha_0 = \arccos(d_0/\Delta T_{12}^{max} \tilde{v}_j) \in (0, \pi/2).$$

Therefore, when $\alpha_0 < \pi/2 \leq \alpha$, $f(\Delta T_{12})$ is a monotonically decreasing function. And it is deduced that $\Delta T_{12}^{max} < \Delta T_{12}^*$ when $\alpha_0 < \alpha \leq \pi/2$. Then $f(\Delta T_{12})$ is also monotonically decreasing when $T_{12} \in (0, \Delta T_{12}^{max}]$. In conclusion, if $\alpha > \alpha_0$, (17) is monotonically decreasing for $\Delta T_{12} \in (0, \Delta T_{12}^{max}]$.

Considering C1 and C2 hold, R_i is able to catch up with R_j before the uncertainty becomes unacceptable. Taking the minimum ΔT_{12} as our objective, the problem is formulated as (18). According to the monotonicity derived above, the problem is induced to solving the equation $v_i^{max} = f(\Delta T_{12})$, whose solution is computed as (21). □

Theorem 1 implicates that when R_i's recovery cost is large (i.e., $\alpha > \alpha_0$), it can only run in faster speed to catch up with R_j. The larger its speed is, the less time it will cost.

Theorem 2. *When C1 and C3 hold, the link between R_i and R_j can be recovered for $\forall v_i \in [v_i^*, v_i^{max}]$.*

1) *The optimal control for minimum chasing time is the same as* (20).
2) *The optimal control for minimum chasing distance is*

$$u_i^* = \tilde{v}_j \tan\alpha \quad (22)$$

from which the minimum chasing distance is given by

$$(v_i \Delta T_{12})_{min} = d_0 \sin\alpha. \quad (23)$$

Proof. The first statement is proved using the same ideas as that of Theorem 1. If $v_i^{max} \geq v_j \tan\alpha$ holds, the minimum chasing distance problem is formulated as (19), whose solution is intuitively obtained from Fig. 3(b), i.e., $d_0 \sin\alpha$. □

Now we turn to situations where the conditions of Theorem 1 and Theorem 2 are not satisfied, i.e., the success of connectivity recovery is not guaranteed.

Theorem 3. *Suppose R_i reconnects with R_j successfully at time $t_0 + T_m$ with certain probability P_r.*

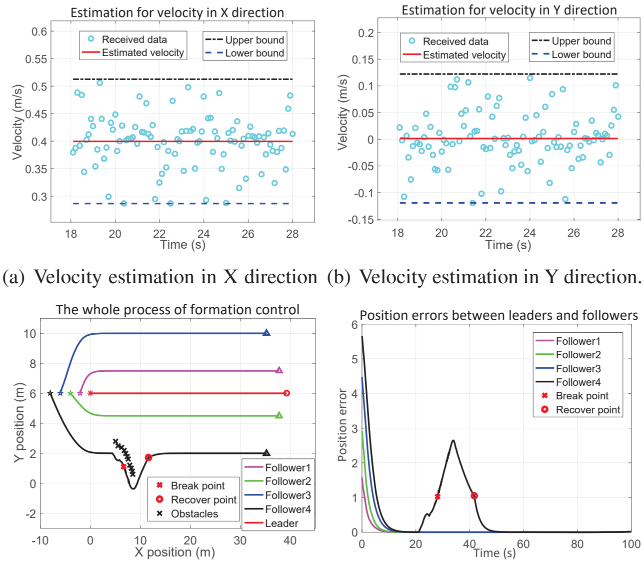
1) *If C1 holds but C2 does not, P_r is given by*

$$P_r = d_c^2/(2M^2 T_m^2), \quad (24)$$

where $T_m (> \Delta T_{12}^{max})$ is the solution of $v_i^{max} = f(T_m)$.

2) *If C1 does not hold, for $v_i^{max} \in (\tilde{v}_j - M, \tilde{v}_j)$*

$$P_r \leq d_c^2/(2M^2 T_m^2), \quad (25)$$



(a) Velocity estimation in X direction (b) Velocity estimation in Y direction.

(c) Follower 4 loses connectivity with follower 2 and recovers it (d) Follower's errors between real position and desired position

Fig. 4. $v_{max} = 1.3$ (m/s), and the root leader's goal position is $[40, 6]$.

where T_m is solution of the following equation

$$[(v_i^{max} + M)T_m - d_c]^2 = d_0^2 + (\tilde{v}_j T_m)^2 - 2 d_0 \tilde{v}_j T_m \cos\alpha.$$

And for $v_i^{max} \in (0, \tilde{v}_j - M)$, we have $P_r = 0$.

Proof. Concerning (24), at time t_2, the communication area of R_i is πd_c^2 while the uncertainty area of R_j $\pi(\sqrt{2} M T_m)^2$, P_r is computed by their area ratio.

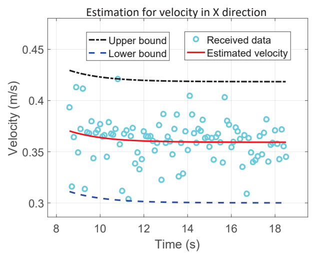
As for (25), when $v_i^{max} < \tilde{v}_j$ but $\tilde{v}_j - M \leq v_i^{max}$, it implicates the uncertainty area of R_j is likely to cover R_i's communication area. In this situation, R_i has a chance to reconnect with R_j, and the maximum probability is obtained when R_i's communication area is just fully covered, i.e., when $P_r = d_c^2/(2M^2 T_m^2)$. If $v_i^{max} < \tilde{v}_j - M$, R_i cannot even move into its estimated uncertainty area, thus $P_r = 0$. □

Theorem 3 depicts the success probability P_r by area ratio. P_r is mainly determined by R_i's velocity constraint v_i^{max}. Given a v_i^{max}, the longer time R_i costs, the smaller P_r is.

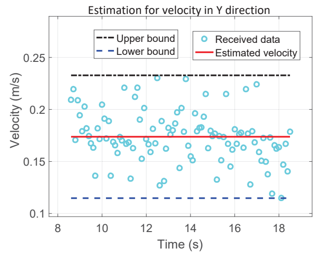
IV. Simulation

In this section, the simulation results are presented to verify the effectiveness of our proposed topology recovery scheme. The formation consists of five robots with communication range $d_c = 6$ m, aiming to form a triangle shape. There is a root leader who determines the movement of whole formation and runs to the goal with $v_0 = 0.4$ (m/s). It is followed by two followers, who also have a follower of their own.

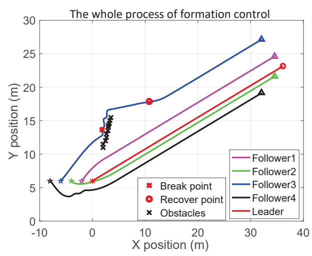
Fig. 4(a)-4(d) are the results for robots with velocity constraint $v_{max} = 1.3$ (m/s). The whole process is illustrated in Fig. 4(c), which consists of three stages. Before t_0, the robots move in different direction to form the specified formation shape. Then, follower 4 encounters obstacles and strats obstacle-avoidance. In this process, the connectivity



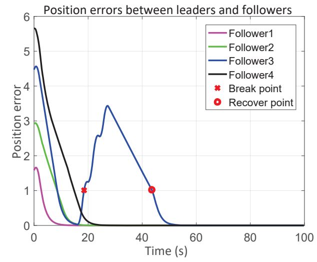
(a) Velocity estimation in X direction



(b) Velocity estimation in Y direction.



(c) Follower 3 loses connectivity with follower 1 and recovers it



(d) Follower's errors between real position and desired position

Fig. 5. $v_{max}=0.6$ (m/s), and the root leader's goal position is $[36, 23]$.

from R_j to R_i is broken at time $t_0 = 28.1s$ and position 'X' point, and obstacle-avoidance is achieved at $t_1 = 33.9s$. After that, follower 4 starts chasing follower 2 based on (18) and the connectivity is recovered at time $t_2 = 41.8s$ and position 'O' point. It is computed that $M = 0.12$ (m/s), $T_c = 35.4s > t_1 - t_0$ and α is an obtuse angle. The successful topology recovery is consistent with Theorem 1.

Fig. 5(a)-5(d) are the results for robots with velocity constraint $v_{max}=0.6$ (m/s). $M=0.06$ (m/s), $t_0 = 18.6s$, $t_1 = 27.2s$, $T_c = 60.6s > t_1 - t_0$, and α is an obtuse angle. Based on Theorem 1, R_i is able to reconnect with R_j. The whole process is similar with last case, but a slight difference is that smaller velocity constraint brings slower convergence speed.

V. Conclusion

In this paper, we investigate the topology recovery problem of formation control for multi mobile robots. Instead of focusing on maintaining connectivity or redesign the topology, we directly propose a connectivity recovery scheme for situations where the communication link has failed due to the influence of the environment. Our method provides a feasible and effective solution for this problem, which consists of motion estimation and control strategy. Considering robots' different velocity constraints and relative position, we obtain two theorems to illustrate the guarantees for the success of topology recovery, respectively. Furthermore, we obtain the success probability when the guarantees are not satisfied. Extensive simulations confirm the effectiveness of the topology recovery scheme. Future directions include exploring more complicated scenarios and establishing a unified mathematical framework.

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