Privacy-preserving Consensus-based Energy Management in Smart Grid

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***Abstract*—This paper investigates the privacy-preserving problem of the distributed consensus-based energy management considering both generation units and responsive demands in smart grid. First, we reveal the private information of consumers including the electricity consumption and the sensitivity of the electricity consumption to the electricity price can be disclosed without any privacy-preserving strategy. Then, we propose a privacy-preserving algorithm to preserve the private information of consumers through designing the secret functions, and adding zero-sum and exponentially decreasing noises. We also prove that the proposed algorithm can preserve the privacy while keeping the optimality of the final state and the convergence performance unchanged. Extensive simulations validate the theoretical results and demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

Furnished with smart meters, smart appliances and distributed energy resources, smart grid brings about the distributed control and energy management [1]. Especially, Distributed Energy Management (DEM) aims to improve the efficiency, scalability and robustness of power grid through the distributed scheduling of both distributed energy resources and responsive consumers by utilizing the information technology [2]. As a vitally important concern of consumers, the private information such as habits and behaviors can be disclosed through the broadcast information during the participation in DEM.

Numerous endeavors have been devoted to designing DEM algorithms in smart grid [3]–[12]. In [3]–[5], DEM problems with different goals are taken into consideration and nonconsensus-based distributed algorithms are designed, which mainly focus on the demand side management (DSM). Compared with these protocols, the consensus-based DEM algorithms can fulfill the global energy management target only through local interactions. Hence, the consensus-based DEM approaches are more flexible, scalable, robust and distributed. Much attention has been paid to the consensus-based economic dispatch problem of smart grid [6], [8]–[10], which only considers the generation side management. In smart grid, considering both of the generation and demand sides in DEM reveals the fairness between consumers and producers, which makes the DEM problem more realistic and meaningful. Authors in [11], [12] proposed a consensus-based DEM algorithm for two sides energy scheduling, which is only effective for undirected connected communication network. Taking the directed communication as well as the transmission losses into consideration, another consensus-based energy management algorithm was developed in [13]. However, all these existing works neglect the privacy concern of the responsive consumers. In [14], the disclosure of generation units' private information, i.e., the final generation power and the parameters of cost functions in the consensus-based economic dispatch process was analyzed, and then authors proposed a privacy-preserving strategy. But the proposed strategy is only effective for the fully undirected communication topology. Furthermore, plentiful achievements for the privacy-preserving demand response through the encryption or aggregation communication design have been attained [15]. However, in the consensus-based DEM, these strategies can lead to much computation and communication burden due to the asymptotically convergence. Meanwhile, differential privacy has been considered as a significant method to preserve the privacy of the distributed optimization problem [16]. But it cannot guarantee that the final solution is the exact optimal one. Therefore, how to design the privacy-preserving consensus-based DEM algorithm considering both of the generation and demand sides under directed communications while guaranteeing the optimality of the final solution is still an open problem.

To deal with the above problem, we first provide the consensus-based DEM algorithm considering both sides and directed communication topology. Then, we investigate how and what kind of the private information of consumers will be disclosed under the algorithm. The privacy-preserving method is designed to conserve the private information through secret functions and adding noises inspired by our previous work [17], under which the optimal solution can be achieved exactly. The main contributions of this paper are as follows.

1. We investigate how the private information including the electricity consumption and the sensitivity of the electricity consumption to the electricity price will be disclosed under the provided consensus-based algorithm.
2. We propose the privacy-preserving consensus-based DEM algorithm (P-CEMA) by adding noises to the broadcast information to preserve the sensitivity and designing secret functions to conserve the electricity consumption.
3. We prove that under P-CEMA, the private information

can be preserved while the convergence and the optimal solution can be maintained.

The remainder of this paper is organized as follows. Section II provides the models and formulations of the problem, and the detailed privacy disclosure is presented in Section III. Section IV provides the privacy-preserving consensus-based DEM algorithm and analyzes the performance. Section V tests the main results through numerical examples and simulations. Conclusion is given in Section VI.

II. Preliminaries and Problem Formulation

A. Network Model

We consider a smart grid with n_g generation units and n_d responsive consumers indexed by $1, 2, \cdots, n_g$ and $n_g + 1, \cdots, n_g + n_d \geq 3$, respectively, where $n = n_g + n_d$. We represent the communication network by a strongly connected (any two nodes are connected by a directed path) directed graph $G = \{V, E\}$ with V being the set of n nodes and $E \subset V \times V$ being the edge set. The unit and consumer sets are expressed by V_g and V_d, respectively, satisfying $V = V_g \cup V_d$. Note that $(j, i) \in E$ if and only if (*iff*) node i can receive information from node j, i.e., node j is the in-neighbor of node i. The in-neighbor set of node i is denoted by $N_i^+ = \{j|(j,i) \in E, j \neq i\}$ and $|N_i^+|$ is the cardinality. Meanwhile, the out-neighbor set of node i is $N_i^- = \{j|(i,j) \in E, j \neq i\}$ and the cardinality is $|N_i^-|$. We do not consider self loops in this paper, i.e., $(i,i) \notin E, \forall i \in V$.

B. Distributed Energy Management

Social welfare maximization not only can maximize the total welfare of the whole society in electricity market, but also can guarantee that each individual's profit is maximized [18]. Therefore, the objective of DEM is to maximize the total social welfare by adjusting the electricity price while balancing the generation and demand. Here, we use price p as a variable to realize coordination among units and consumers. We consider one period DEM problem and introduce the profit functions of units and consumers, respectively.

Profit function of each generation unit: The generation power of each unit $i, i \in V_g$ is denoted by P_i with the lower (upper) bound expressed by P_i^m (P_i^M). The cost of unit i can be represented by the quadratic function of P_i, i.e., $C_i(P_i) = a_i P_i^2 + b_i P_i + c_i$, where a_i, b_i, c_i are fitting parameters. Then, we provide the profit $\pi_i^g(P_i)$ of unit i as

$$\pi_i^g(P_i) = pP_i - C_i(P_i) = -a_i P_i^2 + (p - b_i) P_i - c_i.$$

Profit function of each consumer: The utility function of each consumer $j, j \in V_d$ is denoted by $U_j(P_j)$, which characterizes the satisfaction degree of consumer j consuming P_j. It is usually assumed that $U_j(P_j)$ satisfies the following three properties [19]: 1) The first order derivative $U_j'(P_j)$ satisfies $U_j'(P_j) \geq 0$; 2) The second order derivative $U_j''(P_j)$ satisfies $U_j''(P_j) \leq 0$; 3) $U_j(0) = 0$. Then, the profit function of each consumer j is introduced as $\pi_j^d(P_j) = U_j(P_j) - pP_j$. For each responsive consumer j, it has a certain adjustable range of power (from P_j^m to P_j^M) to maximize its own profit by adjusting P_j for a period due to the excitation of the electricity price.

Therefore, the social welfare, i.e., the sum of profits of all units and consumers, is expressed by,

$$\sum_{i \in V_g} \pi_i^g(P_i) + \sum_{j \in V_d} \pi_j^d(P_j) = \sum_{i \in V_g} -C_i(P_i) + \sum_{j \in V_d} U_j(P_j). \quad (1)$$

We formulate the following optimization problem,

$$\begin{aligned} \min \quad & \sum_{i \in V_g} C_i(P_i) - \sum_{j \in V_d} U_j(P_j) \\ s.t. \quad & \sum_{i \in V_g} P_i = \sum_{j \in V_d} P_j \quad (2a) \\ & P_i^m \leq P_i \leq P_i^M, i \in V. \quad (2b) \end{aligned}$$

where the constraint (2a) describes the balance of the generation and demand, and (2b) are the local power constraints for units and consumers, respectively. It is assumed that the total generation capacity can always meet the demand requirement, i.e., $\sum_{i \in V_g} P_i^m \leq \sum_{j \in V_d} P_j^m \leq \sum_{j \in V_d} P_j^M \leq \sum_{i \in V_g} P_i^M$.

C. Consensus-based Energy Management Algorithm

A row stochastic matrix $W = [w_{ij}]$ and a column stochastic matrix $Q = [q_{ij}]$ are introduced, i.e.,

$$w_{ij} = \begin{cases} \frac{1}{|N_i^+|+1}, j \in N_i^+ \\ 1 - \sum_{j \in N_i^+} w_{ij}, j = i \\ 0, j \notin N_i^+, j \neq i \end{cases}, \quad q_{ij} = \begin{cases} \frac{1}{|N_j^-|+1}, j \in N_i^+ \\ 1 - \sum_{j \in N_i^-} q_{ji}, j = i \\ 0, j \notin N_i^+, j \neq i \end{cases}.$$

Then, we provide the following distributed algorithm to solve problem (2), namely, CEMA [13].

The initialization of $\lambda_i(0)$, $P_i(0)$ and $\xi_i(0)$, $\forall i \in V$ is given as below,

$$\lambda_i(0) = \begin{cases} C_i'(P_i^m), i \in V_g, \\ U_i'(P_i^M), i \in V_d, \end{cases} \quad , P_i(0) = \xi_i(0) = 0, i \in V \quad (3)$$

After initialization, each node broadcasts $\lambda_i(0)$ and $\xi_i(0)$. After receiving all neighbors' information, each node i executes the following iteration process,

$$\lambda_i(k+1) = \sum_{j \in V} w_{ij} \lambda_j(k) + \eta \xi_i(k), i \in V \quad (4a)$$

$$P_i(k+1) = \begin{cases} \underset{P_i^m \leq P_i(k) \leq P_i^M}{\arg\min} [C_i(P_i(k)) - \lambda_i(k+1) P_i(k)], & i \in V_g \\ \underset{P_i^m \leq P_i(k) \leq P_i^M}{\arg\min} [\lambda_i(k+1) P_i(k) - U_i(P_i(k))], & i \in V_d \end{cases} \quad (4b)$$

$$\xi_i(k+1) = \begin{cases} \sum_{j \in V} q_{ij} \xi_j(k) + P_i(k) - P_i(k+1), & i \in V_g \\ \sum_{j \in V} q_{ij} \xi_j(k) + P_i(k+1) - P_i(k), & i \in V_d. \end{cases} \quad (4c)$$

where η is a constant satisfying $0 < \eta < 1$.

Private information of consumers: Here, we are concerned about the private information of consumers, i.e., the quantity of the electricity consumption and the parameters of consumers' utility function which reveals the sensitivity of the electricity consumption to the electricity price. By obtaining the continuous electricity consumption of some consumer for multiple periods, the behaviors and habits of the consumer can be learned. Meanwhile, the sensitivity can be obtained and thus the future behaviors of the consumer can be predicted.

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Although under CEMA, the private information will not be broadcast directly, the private information can still be inferred through the broadcast infromation as the broadcast information is the function of the private information shown in the iteration rule. Hence, it is desirable to preserve the private information under CEMA. Since the iteration rule is based on the consensus, the privacy-preserving average consensus [17] renders the privacy-preserving strategy design for CEMA possible. However, different from the privacy-preserving average consensus, we need to preserve the final state and the functional relationship between variables under CEMA which is much more complex than average consensus.

III. Privacy Disclosure

To analyze the privacy disclosure of CEMA, we first provide the sufficient and necessary condition for the initial state $\xi_i(0)$ and $P_i(0)$, which can guarantee the optimality of the final solution. It can be proved by contradiction and the proof is omitted here due to the space limitation.

***Theorem** 1:* Suppose that the selected η can guarantee the convergence of (4). If and only if

$$\sum_{i\in V}(\xi_i(0)+P_i(0))=0, \tag{5}$$

the final solution is the optimal one of problem (2).

To make the initial states satisfy (5) without any control operator, the simplest and direct method is to set $\xi_i(0)=P_i(0)=0,\forall i\in V$ or $\xi_i(0)=-P_i(0),\forall i\in V$. Under this initialization way, as $\xi_j(0)$ will be broadcast, $P_j(0)$ will be known by the eavesdropper or the neighbors of node i. Then, we provide the following situations in which the private information will be disclosed under CEMA.

1) Node i can observe the neighbor set of its neighbors and $N_j^+\bigcup j\subseteq N_i, j\neq i$, i.e., node i can receive all information of neighbors of node j and node j. Such situation may happen practically, for example, the communication topology is complete.
2) Suppose that there is an eavesdropper, who knows W and Q. Then, the eavesdropper can obtain the private information of consumer j by eavesdropping the information broadcast by node j and neighbors of node j.

The detailed privacy disclosure process is as follows. After initialization and the first time iteration, node i or the eavesdropper obtain $\lambda_j(1)$ through the communication link, and $P_j(1)$ according to

$$\xi_j(1)=\sum_{l\in V}q_{jl}\xi_l(0)+P_j(1)-P_j(0), \tag{6}$$

as $\xi_l(0)$ and $P_j(0), l\in N_j^+\bigcup l=j$ are known. By recursion, $\lambda_j(k), P_j(k), k=1,2,\cdots$ will be disclosed. Through curve fitting, the derivative of the utility function of each consumer and the upper and lower bounds of the power usage can be acquired. Hence, the private information, i.e, the electricity consumption $\lim_{k\to\infty}P_j(k)$ and the sensitivity of the electricity consumption P_i to the electricity price λ_i will be disclosed. Therefore, the interesting problem is how the private information of consumers can be preserved while the convergence and the optimality of the final solution are guaranteed.

IV. Privacy-preserving Algorithm Design

In this section, we first analyze whether there are noises, by adding which to CEMA, we can guarantee the convergence and optimality. Then, we propose a privacy-preserving protocol and analyze how the proposed protocol can preserve the private information of consumers.

A. Noise Adding Analysis

Here we provide the sufficient condition and the necessary condition for CEMA with noises added to the broadcast information $\lambda_i(k)$ and $\xi_i(k),\forall i\in V$. We denote the noise added to $\lambda_i(k)$ and $\xi_i(k)$ by $\vartheta_i(k)$ and $\theta_i(k)$, respectively. The lemmas referred to [20] are as follows.

***Lemma** 1:* If there exists a constant $H\geq$ such that

$$\sum_{k=0}^{\infty}|\vartheta_i(k)|\leq H,\ \sum_{k=0}^{\infty}|\theta_i(k)|\leq H, i\in V, \tag{7}$$

then CEMA achieves asymptotic convergence, i.e.,

$$\lim_{k\to\infty}\lambda_i(k)=\bar{\lambda},\ \lim_{k\to\infty}\xi_(k)=0,\forall i\in V. \tag{8}$$

***Lemma** 2:* If CEMA can achieve asymptotic convergence and the final state is optimal, i.e.,

$$\lim_{k\to\infty}\lambda_i(k)=\lambda^*,\ \lim_{k\to\infty}\xi_(k)=0,\forall i\in V, \tag{9}$$

where λ^* is the optimal price, then there hold

$$\lim_{k\to\infty}\vartheta_i(k)=\lim_{k\to\infty}\theta_i(k)=0,\ \lim_{k\to\infty}\sum_{j=0}^{k}\sum_{i\in V}\theta_i(j)=0. \tag{10}$$

In fact, by adding decreasing and zero-sum noises to each local power mismatch, we cannot preserve the privacy of the electricity consumption. By adding the noise $\theta_i(k)$ to $\xi_i(k+1)$ before broadcasting $\xi_i(k+1)$ to neighbors of node i, we obtain the following equation,

$$\begin{aligned}P_i(k+1)&=\xi_i(k+1)-\sum_{j\in V}q_{ij}\xi_j(k)+P_i(k)-\theta_i(k)\\&=\xi_i(k+1)-\sum_{j\in V}q_{ij}\xi_j(k)-\theta_i(k)+\xi_i(k)\\&\quad-\sum_{j\in V}q_{ij}\xi_j(k-1)+P_i(k-1)-\theta_i(k-1)\\&=\sum_{k}\xi_i(k+1)-\sum_{k}\sum_{j\in V}q_{ij}\xi_j(k)-\sum_{k}\theta_i(k)+P_i(0)\end{aligned} \tag{11}$$

Taking limitations on both sides of (11), one obtains

$$\begin{aligned}&\lim_{k\to\infty}P_i(k+1)\\&=\lim_{k\to\infty}[\sum_{k}\xi_i(k+1)-\sum_{k}\sum_{j\in N_i^+\cup i}q_{ij}\xi_j(k)-\sum_{k}\theta_i(k)+P_i(0)]\end{aligned} \tag{12}$$

Since $P_i(0)$ is known and $\xi_j(k), j\in N_i^+\cup i, k=1,2,\cdots$ can be heard through communication links, $\lim_{k\to\infty}P_i(k)$ can be obtained if the sum of the added noises is zero. However, specific $P_i(k),\forall k=1,2,\cdots$ cannot be obtained, which means that the sensitivity of P_i to λ_i cannot be inferred. Hence, in this way, only part of the private information can be preserved.

B. P-CEMA

Inspired by [17], we propose the privacy-preserving algorithm to preserve the complete privacy of the consumer while keeping the optimality. Differently, we preserve the final state and the functional relationship between two states of CEMA, which is more complex than average consensus considered in [17]. To avoid the disclosure of the private electricity consumption, we introduce a secret continuous function $F_{ij}(z): R \to R, i \in N_j^+$ for node i with respect to its out-neighbor node j. Suppose that $F_{ij}(z)$ is only available to node i and j, which can make the initial state $P_i(0)$ not inferrable specifically. To guarantee the convergence and optimality, we design the noise adding process for $\lambda_i(k)$ and $\xi_i(k), \forall i \in V$, where the absolute bound of the noise is exponentially decreasing with iterations and the sum of all noises at all iterations will converge to zero. The details are shown in Algorithm 1.

Algorithm 1 Privacy-preserving CEMA (P-CEMA)

Initialization:

1. Each node i selects a uniform distribution random variable $\nu_i(0)$ from interval $[-\frac{1}{\sqrt{3}\sigma}, \frac{1}{\sqrt{3}\sigma}]$, where $\sigma > 0$ is a constant, and arbitrarily gives constant sequences z_{ij} ($\in R$) for $j \in N_i^-$. Meanwhile, node i initializes $\lambda_i(0)$, $\xi_i(0)$ and $P_i(0)$ according to (3) and sets an appropriate η and the termination errors ϵ_m and ϵ_l.
2. Each node i sets $\theta_i(0) = \nu_i(0)$ and transmits z_{ij}, $\lambda_i(0)$ and $\xi_i(0)$ to its neighboring node j.
3. Each node i calculates $\tilde{\nu}_i(0)$ by,

$$\tilde{\nu}_i(0) = \nu_i(0) - [\sum_{j\in N_i^-} F_{ij}(z_{ij}) - \sum_{j\in N_i^+} F_{ji}(z_{ji})], \forall i \in V. \tag{13}$$

Iteration: loop

4. Each node i generates a uniform distribution random variables $\nu_i(k)$ from interval $[-\frac{1}{\sqrt{3}\sigma}, \frac{1}{\sqrt{3}\sigma}]$ for $k \geq 1$. Then, node i updates $\theta_i(k)$ according to,

$$\theta_i(k) = \begin{cases} \varrho\nu_i(1) - \tilde{\nu}_i(0) & \text{if } k = 1 \\ \varrho^k \nu_i(k) - \varrho^{k-1}\nu_i(k-1) & \text{if } k \geq 2 \end{cases} \tag{14}$$

where $\varrho \in (0, 1)$ is a constant for all nodes.

5. Each node i updates $\lambda_i(k+1)$, $P_i(k+1)$ and $\xi_i(k+1)$ according to (4) and adds the noise $\theta_i(k)$ to $\xi_i(k+1)$, i.e., $\xi_i(k+1) = \xi_i(k+1) + \theta_i(k)$. Then, node i broadcasts $\lambda_i(k+1)$ and $\xi_i(k+1)$ to its neighbors.
6. If $|\xi_i(k)| \leq \epsilon_m, \forall i \in V$ and $|\lambda_i(k) - \lambda_i(k-1)| \leq \epsilon_l, \forall i \in V$, break.

Output: $\lambda_i(k), P_i(k), \forall i \in V$.

C. Performance Analysis of P-CEMA

We prove that P-CEMA can preserve the privacy of the electricity consumption and the sensitivity of the electricity consumption to the electricity price, while guaranteeing the convergence and optimality.

Theorem 2: Under P-CEMA, there exists a sufficiently small η such that $\lim_{k\to\infty} P_i(k) = P_i^*, \forall i \in V$, where $P_i^*, \forall i \in V$ is the optimal solution, while P_i^* and the sensitivity of P_i to $\lambda_i, \forall i \in V$ can be preserved.

Proof: To prove the optimality, by adding $\theta_i(k)$ for $k = 1, 2, \cdots, i = 1, \cdots, n$, we will have,

$$\sum_{k=1}^{\infty}\sum_{i\in V} \theta_i(k) = \sum_{i\in V}\sum_{j\in N_i^+} F_{ji}(z_{ji}) - \sum_{i\in V}\sum_{j\in N_i^-} F_{ij}(z_{ij}). \tag{15}$$

As the communication network is strongly connected, there holds

$$\sum_{i\in V}\sum_{j\in N_i^+} F_{ji}(z_{ji}) - \sum_{i\in V}\sum_{j\in N_i^-} F_{ij}(z_{ij}) = 0. \tag{16}$$

From (16), we note that the sum of all noises added to $\xi_i(k), \forall i \in V, k = 1, \cdots$ is zero and the noise added to $\xi_i(k)$ is exponentially decreasing and goes to zero with k. Hence, according to Lemma 1 and 2, the convergence and optimality of P-CEMA can be guaranteed.

We then prove that $P_i^*, i \in V$ will not be disclosed specifically. According to P-CEMA, for each node i, there holds,

$$\begin{aligned} \lim_{k\to\infty}\sum_k \theta_i(k) &= \nu_i(0) + \varrho\nu_i(1) - \tilde{\nu}_i(0) + \varrho^2\nu_i(2) - \varrho\nu_i(1) + \cdots \\ &= \nu_i(0) - \nu_i(0) + [\sum_{j\in N_i^-} F_{ij}(z_{ij}) - \sum_{j\in N_i^+} F_{ji}(z_{ji})] \\ &= [\sum_{j\in N_i^-} F_{ij}(z_{ij}) - \sum_{j\in N_i^+} F_{ji}(z_{ji})] \end{aligned} \tag{17}$$

Since $F_{ij}(z_{ij})$ is only available to node i and j, neighbors of node i or eavesdropper cannot know the specific value of $[\sum_{j\in N_i^-} F_{ij}(z_{ij}) - \sum_{j\in N_i^+} F_{ji}(z_{ji})]$. Therefore, neighbors of node i or eavesdroppers cannot compute $\lim_{k\to\infty} P_i(k)$ exactly according to (12), implying that the final electricity consumption of each consumer i will not be disclosed.

Then, we prove that the sensitivity of the electricity consumption to the price will not be known explicitly. Due to (11) and $P_i(0) = \xi_i(0) = 0, i \in V$, we have

$$P_i(1) = \xi_i(1) - \sum_{j\in N_i} q_{ij}\xi_j(0) + P_i(0) - \theta_i(0) = \xi_i(1) - \theta_i(0) \tag{18}$$

As the exact value of $\theta_i(0)$ is unknown, $P_i(1)$ cannot be fixed by obtaining $\xi_j(1), j \in N_i^+ \cup i$. For the same reason, $P_i(k), k = 1, 2, \cdots$ cannot be obtained. Meanwhile, since $\lim_{k\to\infty} P_i(k)$ is not known, $P_i(k), k = 1, 2, \cdots$ cannot be fixed backwards. Although we obtain specific $\lambda_i(k), k = 1, 2, \cdots$, the sensitivity of P_i to λ_i cannot be acquired. ■

Remark 1: As we can see that from P-CEMA, the secret function is only required at the initialization of the noise $\theta_i(0), \forall i \in V$. Compared with encrypting communication variables, utilizing secret function is more simple.

V. Simulations

In this section, we illustrate the performance of the proposed privacy-preserving algorithm by extensive simulation results.

We consider the IEEE-39 Bus System with $n_g = 10$ generation units and $n_d = 18$ consumers, where the communication topology is strongly connected with directed edges. The utility function of each consumer j is described by

$$U_j(P_j) = \begin{cases} \omega_j P_j - \alpha_j P_j^2, & P_j \leq \frac{\omega_j}{2\alpha_j} \\ \frac{\omega_j^2}{4\alpha_j}, & P_j > \frac{\omega_j}{2\alpha_j} \end{cases}. \tag{19}$$

The parameters of cost functions of generation units and utility functions of consumers are referred by [13]. Meanwhile, let $\eta = 1.786 * 10^{-3}$, $\sigma = 0.1732$, $\varrho = 0.03$. The secret function is given as $F_{ij}(z_{ij}) = j * z_{ij} + i$.

First, we investigate the convergence and optimality performance of P-CEMA compared with that of CEMA. We set all z_{ij}'s from the random range [0,5]. It can be observed from Fig. 1 that P-CEMA can achieve almost the same convergence

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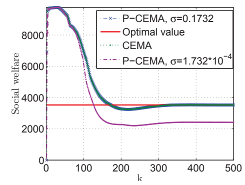


Fig. 1. Convergence and optimality of P-CEMA

speed compared with CEMA and the optimality can always be guaranteed. By adding noises with a sufficiently large σ, we find that the convergence speed will remain unchanged. When σ is too much small, i.e., the noise will be selected from a large interval, P-CEMA may not reach convergence.

Then, we investigate how the private information can be preserved under P-CEMA with same setting of z_{ij}'s. It can be observed from Fig. 2 that for different kinds of generation units and responsive consumers, the final power and the sensitivity can not be inferred by neighbors or the eavesdropper without knowing the secret function. Hence, the privacy of consumers is preserved under the proposed algorithm while the convergence and optimality remained unchanged. To investigate how the different distribution will affect the performance of the algorithm, we set z_{ij} as $z_{ij} = 5 * i/j$ and compare the performance of P-CEMA with the normal and uniform distributions and different secret functions. It can be seen in Fig. 4(a) that different noises will not affect the performance of P-CEMA differently, while different secret functions will induce more differences shown in Fig. 4(b).

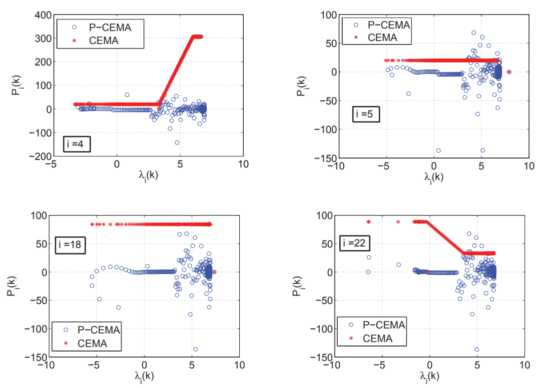
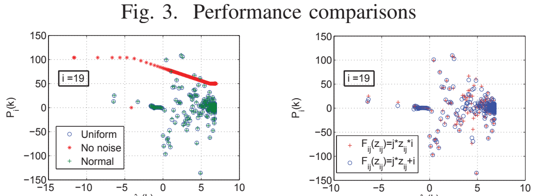


Fig. 2. Units and consumers

VI. Conclusion

In this paper, we mainly consider the privacy-preserving problem of the consensus-based algorithm for DEM considering multiple generation units and responsive consumers. We reveal that the private information of consumers including the electricity consumption and the sensitivity of the electricity consumption to the electricity price can be disclosed from the considered algorithm. To preserve the privacy of the consumers, the secret function is utilized to preserve the quantity of electricity consumption and the random and exponentially decreasing noise is exploited to preserve the utility function inspired by our works in [17]. Illustrating examples and extensive simulation results verify the effectiveness of P-CEMA and the theoretical analysis.

Fig. 3. Performance comparisons



(a) Different distribution noises (b) Different secret functions

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