Consensus-Based Energy Management in Smart Grid With Transmission Losses and Directed Communication

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***Abstract*—This paper investigates the problem of distributed energy management for both generation and demand side in smart grid. Different from existing works, we formulate a social welfare maximization problem for a more practical scenario by taking transmission losses into account. The formulated problem is non-convex due to the non-convexity of the power balance equality constraint caused by the transmission losses. To solve the problem, we first transform the equality constraint into an inequality constraint and obtain a new convex optimization problem. We then derive a sufficient condition to guarantee that the new problem has the same solution as the original one. Because of the coupling in the constraint, Lagrange duality method is adopted to decompose the problem. Considering the general communication topology among generators and demands, i.e., directed connected topology, we design a consensus-based algorithm to solve the problem in a distributed way. We also prove the convergence and optimality of the proposed algorithm, under which the social welfare maximization is achieved. Extensive simulations validate the theoretical results and demonstrate the effectiveness of the proposed algorithm.**

***Index Terms*—Distributed energy management, transmission losses, social welfare maximization, consensus, smart grid.**

I. Introduction

A. Motivation

THE NEW envisioned power grid, namely smart grid, is integrated with smart infrastructure, advanced management and intelligent protection technology. It is much more clean, reliable, safe, resilient, efficient and sustainable than the traditional power grid [1]. Smart grid is equipped with intelligent controllable electrical devices and advanced communication network, which makes the distributed control and distributed energy management possible [2]. Distributed energy management (DEM) is to monitor, control and optimize the performance of power grid in a distributed way by using information technology [3]. It plays a key role in optimization and scheduling of both generators and demands in smart grid. With distributed energy management algorithms, the power grid will be more scalable and robust, and less operation information will be required.

Transmission and distribution losses are estimated at about 6% each year, where 10% losses are transmission line losses [4]. The national electricity consumption of China estimated by China Electricity Council will reach the range of $6,020$ billion-$6,610$ billion kWh in 2015 [5], which means that the total transmission losses are about 36.12 million-39.66 million kilowatts. It indicates that transmission losses are huge and cannot be ignored. Furthermore, transmission losses need to be considered in the operation of power network [6]. Hence, it is necessary to take transmission losses into consideration for energy management. Transmission losses are also significant for the accurate formulation of energy management problem [7]. The existence of transmission losses induces different variations of incremental cost for different generations. A distributed consensus-based algorithm was also proposed for economic dispatch considering transmission losses in [8], which only considers generation side.

Multi-agent system is promising for distributed energy management of future smart grid [2]. In multi-agent system, it is important to perform coordinated tasks in a distributed way while the interagent communication costs are limited. The communication cost of protocols with directed information flow is smaller than that with undirected communication topology [9]. Besides, with integration of smart meters and controllable electronic devices, distributed energy management strategy for smart grid should be highly scalable [10]. Considering the directed communication can strengthen the scalability of distributed energy management. In communication network, there are inevitable factors leading to directed communication due to the packet loss and communication interference [11]–[13]. Hence, it is necessary and meaningful to consider the directed communication.

Manuscript received September 9, 2015; revised November 8, 2015 and December 15, 2015; accepted December 16, 2015. Date of publication January 20, 2016; date of current version August 21, 2017. This work was supported in part by the National Program for Special Support of Top Notch Young Professionals, in part by the National Science Foundation of China under Grant U1401253 and Grant 61503332, in part by the China Postdoctoral Science Foundation under Grant 2015T80614, and in part by the State Key Laboratory of Synthetical Automation for Process Industries of Northeastern University. Paper no. TSG-01105-2015. *(Corresponding author: Jiming Chen.)*

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Digital Object Identifier 10.1109/TSG.2015.2513772

B. Related Works

Many efforts have been devoted to realizing distributed energy management in smart grid [8], [14]–[26]. In [14]–[18], distributed energy management problem is considered and nonconsensus-based distributed algorithms are designed which mainly focus on the demand side management (DSM). Li *et al.* proposed a decoupling algorithm for responsive demands management where the utility company has to provide the price information [14]. A distributed algorithm was proposed for distributed DSM and applied in plug-in hybrid electric vehicles in [15], where the price information based on the global system load is needed. In [16]–[18], game theory based DSM protocols are provided. In [16], it is assumed that each player has to communicate with all others. In [17] and [18], it is assumed that there exits the utility company. Compared with these nonconsensus-based DEM protocols, consensus-based algorithms can realize global energy management target only with local communication. Hence, consensus-based DEM approaches are more flexible, scalable, robust and distributed. As a branch of distributed computing, consensus has been widely used in economic dispatch problem of smart grid in [19]–[22], where only distributed generation side management is considered for synchronous communication networks. Considering the asynchronous communication, Mohammadi *et al.* [23] developed a novel distributed approach for DC optimal power flow to reduce communication overhead. In smart grid, considering both sides in distributed energy management reveals fairness between consumers and producers and the whole distributed energy management problem becomes more practical and meaningful. Vale *et al.* [24] proposed a multi-player based energy management algorithm where distributed energy resources and demand response are considered. Under this algorithm, each player has to know all other players' states and thus it cannot be realized in a distributed way. In [25] and [26], a consensus-based distributed energy management algorithm for both sides was proposed for undirected connected communication network. However, the strategy is only effective when the communication network is undirected connected and transmission losses are ignored.

C. Contributions

To solve the above problem, we formulate a distributed energy management problem to maximize social welfare. Both transmission losses and directed connected networks are considered in the problem. Then, we design a fully distributed algorithm to solve the optimization problem. The main contributions of this paper are as follows.

1. A practical energy management optimization problem based on social welfare maximization is formulated for smart gird. We consider both generation and demand sides as well as transmission losses. The problem is a challenging non-convex problem due to the non-convex power balance constraint.
2. We firstly convert the original problem to a convex optimization problem by relaxing the non-convex equality constraint. Then, a sufficient condition is provided to guarantee that the new problem has the same optimal solution as the original one.
3. Considering the directed connected synchronous network, we design a Consensus-based Energy Management Algorithm (CEMA) to solve the new convex problem. In CEMA, ratio consensus is adopted to balance generation and demand. We theoretically prove the convergence and optimality of CEMA and illustrate them by simulations.

The remainder of this paper is organized as follows. Section II provides the preliminaries and formulates the problem. The problem is analyzed and converted in Section III. Section IV decouples the convex problem and gives consensus-based energy management algorithm (CEMA). Section V analyzes the performance of CEMA. The main results are examined through extensive simulations in Section VI. Conclusion is given in Section VII.

II. Preliminaries and Problem Formulation

A. Network Model

Consider a power grid with $\mathcal{N}_g$ distributed generators and $\mathcal{N}_d$ responsive demands indexed by $1, 2, \cdots, \mathcal{N}_g$ and $\mathcal{N}_g+1, \cdots, \mathcal{N}$, respectively, where $\mathcal{N} = \mathcal{N}_g+\mathcal{N}_d$. A directed connected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ is used to represent the communication topology of the network, where $\mathcal{V}$ is the set of $\mathcal{N}$ nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set. The generators set and the demands set are denoted as $\mathcal{V}_g$ and $\mathcal{V}_d$, respectively, and $\mathcal{V} = \mathcal{V}_g \bigcup \mathcal{V}_d$. It is noted that $(j, i) \in \mathcal{E}$ if and only if (*iff*) node i can receive information from node j, i.e., node j is the in-neighbor of node i. The in-neighbor set of node i is defined as $\mathcal{N}_i^+ = \{j|(j, i) \in \mathcal{E}, j \neq i\}$ and $|\mathcal{N}_i^+|$ is the cardinality. The out-neighbor set of node i is defined as $\mathcal{N}_i^- = \{j|(i, j) \in \mathcal{E}, j \neq i\}$ and $|\mathcal{N}_i^-|$ is the cardinality. Self loop is not considered here, i.e., $(i, i) \notin \mathcal{E}, \forall i \in \mathcal{V}$. It should be pointed out that in this paper, we only assume that the communication network is strongly connected, i.e., any two nodes are connected by a directed path. This assumption relaxes the undirected connection assumption in [26] and renders the distributed energy management problem more general.

B. Distributed Energy Management

In smart grid, distributed energy management aims to improve the efficiency of electricity usage and guarantee fairness among all participants (both generators and the demands) under negotiation [6]. Electricity price is an important variable concerned by both sides. Real time pricing is used to improve the efficiency of resource usage and smooth the load curve for smart grid in demand response [27]–[29]. Here, we use price as a variable to realize coordination among generators and demands. The objective is to maximize the total social welfare by adjusting acceptable price and the generation power or demand power. The balance between generators and the demands is required simultaneously since it plays a key role in stable operation of power grid.

The details are given in the following. Dividing a day into $\mathcal{T}$ time periods and the length of each period is constant (for example, one hour). In each period $t, t \in \{1, 2, \cdots, \mathcal{T}\}$, the electricity price is denoted by $p(t)$. By referring to [14], the multiple times demand response can be decoupled as multiple sub-problems. Therefore, we only consider one period energy management problem and to simplify the problem formulation, we omit all t for the following referred variables (for example, p denotes $p(t)$). Through communication, each generator or responsive demand aims to maximize its profit when the generation and demand power is balanced. Then, we introduce the profit functions for generators and demands respectively.

1) *The Profit Function of the Generator:* For each generator $i, i \in \mathcal{V}_g$, its generation power is represented as $\mathcal{P}_i$ with a lower (upper) bound denoted by $\mathcal{P}_i^m$ ($\mathcal{P}_i^M$). Referring [19]–[22], the cost $\mathcal{C}_i(\mathcal{P}_i)$ of generator i can be denoted as the quadratic function of $\mathcal{P}_i$, which means

$$\mathcal{C}_i(\mathcal{P}_i) = a_i\mathcal{P}_i^2 + b_i\mathcal{P}_i + c_i,$$

where a_i, b_i, c_i are fitting parameters of the cost function for generator i. In power grid, since the transmission losses are inevitable, modeling this factor into its profit function is important and practical. According to the micro-incremental transmission losses of each generator [30], the amount of transmission losses $\mathcal{P}_i^{\mathcal{L}}$ induced by generator i can be represented as the simple quadratic function. Hence, we have,

$$\mathcal{P}_i^{\mathcal{L}} = \mathcal{B}_i\mathcal{P}_i^2,$$

where $\mathcal{B}_i$ is the losses coefficient and satisfies $0 \le \mathcal{B}_i < a_i$. Therefore, the profit $\pi_i^g(\mathcal{P}_i)$ of generator i can be represented as

$$\begin{aligned}\pi_i^g(\mathcal{P}_i) &= p\Big(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\Big) - \mathcal{C}_i(\mathcal{P}_i)\\ &= p\Big(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\Big) - \Big(a_i\mathcal{P}_i^2 + b\mathcal{P}_i + c_i\Big)\\ &= -(p\mathcal{B}_i + a_i)\mathcal{P}_i^2 + (p - b_i)\mathcal{P}_i - c_i.\end{aligned}$$

2) *The Profit Function of Each Demand:* For each demand $j, j \in \mathcal{V}_d$, its utility function is written as $\mathcal{U}_j(\mathcal{P}_j)$. $\mathcal{U}_j(\mathcal{P}_j)$ represents the satisfaction degree for the electricity usage $\mathcal{P}_j$. $\mathcal{U}_j(\mathcal{P}_j)$ is usually assumed to satisfy the following three properties [27]:

- The first order differential denoted by U_j' satisfies $\mathcal{U}_j'(\mathcal{P}_j) \ge 0$, i.e., it is a nondecreasing function.
- The second order differential denoted by U_j'' satisfies $\mathcal{U}_j''(\mathcal{P}_j) \le 0$, i.e., utility will get saturated.
- $\mathcal{U}_j(0) = 0$, i.e., without using electricity, the satisfaction is zero.

The following is an example of utility function [27],

$$\mathcal{U}_j(\mathcal{P}_j) = \begin{cases} \omega_j\mathcal{P}_j - \alpha_j\mathcal{P}_j^2, & \mathcal{P}_j \le \dfrac{\omega_j}{2\alpha_j} \\ \dfrac{\omega_j^2}{4\alpha_j}, & \mathcal{P}_j > \dfrac{\omega_j}{2\alpha_j} \end{cases} \tag{1}$$

Then, the profit function of each demand j is introduced as

$$\pi_j^d(\mathcal{P}_j) = \mathcal{U}_j(\mathcal{P}_j) - p\mathcal{P}_j.$$

For each responsive demand, it has a certain adjustable range of power to maximize its own profit by adjusting $\mathcal{P}_j$ during a period because of the generators' excitation. For example, when generators set different prices for different periods, the residential consumer will adjust the power consumption to lower price period.

C. Optimization Problem

Considering the selfishness of each participant, each generator or demand aims to maximize its own profit, which may lead to the coordination failure. Social welfare maximization not only can maximize the total welfare of the whole society in electricity market, but also can guarantee that each individual profit is maximized [31]. Hence, both efficiency and fairness are considered. Social welfare maximization is widely used in energy management of smart grid [14], [26]. Here, we use the social welfare maximization model to realize the coordination among participants. The social welfare, i.e., the sum profit of the generators and the demands, is given as,

$$\begin{aligned}&\sum_{i\in\mathcal{V}_g}\pi_i^g(\mathcal{P}_i) + \sum_{j\in\mathcal{V}_d}\pi_j^d(\mathcal{P}_j)\\ &\quad= \sum_{i\in\mathcal{V}_g}\Big[p\Big(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\Big) - \mathcal{C}_i(\mathcal{P}_i)\Big] + \sum_{j\in\mathcal{V}_d}\big[\mathcal{U}_j(\mathcal{P}_j) - p\mathcal{P}_j\big]\\ &\quad= \sum_{i\in\mathcal{V}_g} -\mathcal{C}_i(\mathcal{P}_i) + \sum_{j\in\mathcal{V}_d}\mathcal{U}_j(\mathcal{P}_j)\\ &\qquad+ \sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\Big) - \sum_{j\in\mathcal{V}_d}\mathcal{P}_j.\end{aligned} \tag{2}$$

Since the power generated and demanded should be balanced for the reliable operation of the power grid [30], i.e., $\sum_{i\in\mathcal{V}_g}(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2) = \sum_{j\in\mathcal{V}_d}\mathcal{P}_j$, (2) can be simplified as,

$$\sum_{i\in\mathcal{V}_g}\pi_i^g(\mathcal{P}_i) + \sum_{j\in\mathcal{V}_d}\pi_j^d(\mathcal{P}_j) = \sum_{i\in\mathcal{V}_g} -\mathcal{C}_i(\mathcal{P}_i) + \sum_{j\in\mathcal{V}_d}\mathcal{U}_j(\mathcal{P}_j). \tag{3}$$

Hence, we formulate the following optimization problem,

$$\begin{aligned}\min\quad & \sum_{i\in\mathcal{V}_g}\mathcal{C}_i(\mathcal{P}_i) - \sum_{j\in\mathcal{V}_d}\mathcal{U}_j(\mathcal{P}_j)\\ s.t.\quad & \sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\Big) = \sum_{j\in\mathcal{V}_d}\mathcal{P}_j && \text{(4a)}\\ & \mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M, i \in \mathcal{V}_g && \text{(4b)}\\ & \mathcal{P}_j^m \le \mathcal{P}_j \le \mathcal{P}_j^M, j \in \mathcal{V}_d. && \text{(4c)}\end{aligned}$$

where the constraint (4a) describes the total generation and total demand power balance, (4b) and (4c) are the local power constraints for each generator and each demand, respectively.

Note that in power grid, the generated electrical power should always satisfy the requirement of the total demand power. Hence, it is reasonable to assume that the total generation capacity meet the demand requirement, i.e., there holds,

$$\sum_{i\in\mathcal{V}_g}\mathcal{P}_i^m \le \sum_{j\in\mathcal{V}_d}\mathcal{P}_j^m \le \sum_{j\in\mathcal{V}_d}\mathcal{P}_j^M \le \sum_{i\in\mathcal{V}_g}\mathcal{P}_i^M. \tag{5}$$

In problem (4), only the first constraint is a global equality constraint and it is an equality quadratic constraint, which leads to the non-convex feasible set. It follows from [32] and [33] that the original problem is not a convex optimization problem due to non-convex constraint (4a) and thus difficult to be solved directly. To solve the problem, detailed analysis will be provided in the following section.

III. Problem Analysis

In this section, to solve problem (4), we firstly convert it to a convex problem by relaxing the constraint (4a). Then, we provide a sufficient condition regarding the generation and the demand capacity, which can be satisfied in practical scenarios. The condition can guarantee that the converted problem has the same optimal solution as the original one. We also give the rigorous proof of the sufficient consideration by contradiction.

A. Problem Transformation

In order to solve the above problem, we firstly convert the equality constraint into an inequality one. Then, problem (4) is transformed to (6). Problem (6) is a strictly convex problem since both the objective function and the constraints are convex.

$$\begin{aligned} \min \quad & \sum_{i\in\mathcal{V}_g} \mathcal{C}_i(\mathcal{P}_i) - \sum_{j\in\mathcal{V}_d} \mathcal{U}_j(\mathcal{P}_j) \\ s.t. \quad & \sum_{i\in\mathcal{V}_g} \left(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\right) \geq \sum_{j\in\mathcal{V}_d} \mathcal{P}_j \qquad (6a) \\ & (4b) \text{ and } (4c). \end{aligned}$$

B. A Sufficient Condition

In the following theorem, we give a sufficient condition to guarantee that problem (4) and (6) have the same solution.

Theorem 1: Suppose that the generation capacity and the demand capacity satisfy

$$\sum_{j\in\mathcal{V}_d} \mathcal{P}_j^M \geq \sum_{i\in\mathcal{V}_g} \left(\mathcal{P}_i^m - \mathcal{B}_i\left(\mathcal{P}_i^m\right)^2\right). \qquad (7)$$

The optimal solution of problem (6), denoted by $\mathcal{P}^*$, satisfies

$$\sum_{j\in\mathcal{V}_d} \mathcal{P}_j^* = \sum_{i\in V_g} \left(\mathcal{P}_i^* - \mathcal{B}_i\left(\mathcal{P}_i^*\right)^2\right),$$

i.e., problem (4) and (6) have the same optimal solution.

To prove the equivalence of problem (6) and (4) under condition (7), we analyze the KKT conditions of the convex optimization problem (6) by referring to [32]. We firstly consider the inequivalent case, i.e., constraint (6a) always takes the greater-than sign. Through the analysis of KKT conditions, we get the result which contradicts condition (7). It is proved that the optimal solution is always achieved for problem (6) when (6a) takes the equal sign, which means that the two problem (6) and (4) are equivalent. The detailed proof can be seen in Appendix A.

Remark 1: It is noted that when the maximal total demand power is larger than or equal to the total minimal generation power subtracting power losses, the converted convex problem always achieves its optimal solution with (6a) obtaining the equal sign. It means that the non-convex optimization problem (4) can be converted to a convex optimization problem. In practical scenarios, the total minimal generation power can be zero, which means that the sufficient condition can always be satisfied. Here, we assume that each generator unit has a lower bound, which guarantees the profit of each generator will be strictly greater than zero. Furthermore, the sum of these lower bounds subtracting power losses will be less than or equal to the total demand power. Hereafter, we just need to solve the convex optimization problem (6) to achieve the optimal solution of the original problem.

IV. Consensus-Based Algorithm Design

In the above section, we convert the non-convex optimization problem to a convex one and provide a sufficient condition under which two problems are equivalent. To solve the converted problem (6) in a distributed way, we firstly decouple it by typical Lagrange approach and then give some basic concepts regarding consensus and ratio consensus. Then, we introduce the new variable to denote the local power mismatch, which is important for the convergence of global power mismatch. Considering the directed communication network, we provide consensus-based iteration rule to guarantee the convergence of incremental cost (incremental utility). Meanwhile, we utilize ratio consensus-based iteration rule to guarantee the convergence of power mismatch. The detailed description can be found in the Consensus-based Energy Management Algorithm (CEMA).

A. Problem Decoupling

In problem (6), the objective function is the sum of local functions, and constraint (4b) and (4c) are local and constraint (6a) is a global coupled constraint. Here, we can decouple problem (6) into multiple sub-optimization problems if constraint (6a) is decoupled. We transfer the constraint (6a) to the objective function and the regarding Lagrangian function can be written as

$$\begin{aligned} \mathcal{L}(\mathcal{P}, \lambda) = & \sum_{i\in\mathcal{V}_g} \mathcal{C}_i(\mathcal{P}_i) - \sum_{j\in\mathcal{V}_d} \mathcal{U}_j(\mathcal{P}_j) \\ & + \lambda\left(\sum_{j\in\mathcal{V}_d} \mathcal{P}_j - \sum_{i\in\mathcal{V}_g}\left(\mathcal{P}_i - \mathcal{B}_i\mathcal{P}_i^2\right)\right), \qquad (8) \end{aligned}$$

where $\mathcal{P} = [\mathcal{P}_1, \cdots, \mathcal{P}_N]'$ and $\lambda \geq 0$ is the lagrange multiplier. From (8), one can decouple the original optimization problem (6) into $\mathcal{N}$ sub-optimization problems with local constraints for a given λ.

By referring to [19], we define λ_i as incremental cost (incremental utility) for the generator (the demand) i as

$$\lambda_i = \begin{cases} \dfrac{\mathcal{C}_i'(\mathcal{P}_i)}{(1 - 2\mathcal{B}_i\mathcal{P}_i)}, & i \in \mathcal{V}_g \\ \mathcal{U}_i'(\mathcal{P}_i), & i \in \mathcal{V}_d \end{cases} \qquad (9)$$

If each $\mathcal{P}_i, i \in \mathcal{V}$ satisfies its local constraint, (6a) holds, and all $\lambda_i, \forall i \in \mathcal{V}$ achieve consensus, i.e., $\lambda_i = \lambda^* > 0$, $\forall i \in \mathcal{V}$, the optimization problem (6) is solved. Variables $\lambda_i, \forall i \in \mathcal{V}$ need to converge and the global power mismatch has to converge to zero. Hence, consensus theory could be used to solve problem (6) in a distributed way.

We will discuss the details regarding the derivation of (9), which is important for the following proposed algorithm and the theoretical analysis. For $i \in \mathcal{V}_g$, since that $\mathcal{C}_i(\mathcal{P}_i)$ is a quadratic function, we have that the derivation of λ_i must be positive according to (10). It means that λ_i is a monotonically increasing function of $\mathcal{P}_i$ when $i \in \mathcal{V}_g$. For $i \in \mathcal{V}_d$, we have $\lambda_i' = \mathcal{U}_i''(\mathcal{P}_i)$. According to the properties of $\mathcal{U}_i(\mathcal{P}_i)$ provided in Section II, there holds $\lambda_i' = \mathcal{U}_i''(\mathcal{P}_i) \leq 0$. We can see that λ_i is a non-increasing function of $\mathcal{P}_i$ when $i \in \mathcal{V}_d$.

$$\lambda_i' = \left(\frac{2a_i\mathcal{P}_i + b_i}{1 - 2\mathcal{B}_i\mathcal{P}_i}\right)' = \frac{a_i + b_i\mathcal{B}_i}{2\mathcal{B}_i^2\left(\mathcal{P}_i - \frac{1}{2\mathcal{B}_i}\right)^2} > 0, i \in \mathcal{V}_g. \tag{10}$$

B. Preliminaries of Consensus

Consensus is a popular tool in distributed computation and has attracted much attention [34]–[37]. We introduce a row stochastic matrix $\mathcal{W} = [w_{ij}]$ and a column stochastic matrix $\mathcal{Q} = [q_{ij}]$. The elements are represented as,

$$w_{ij} = \begin{cases} \dfrac{1}{|\mathcal{N}_i^+| + 1}, & j \in \mathcal{N}_i^+ \\ 1 - \sum\limits_{j \in \mathcal{N}_i^+} w_{ij}, & j = i \\ 0, j \notin \mathcal{N}_i^+, & j \neq i \end{cases} \tag{11}$$

$$q_{ij} = \begin{cases} \dfrac{1}{\left|\mathcal{N}_j^-\right| + 1}, & j \in \mathcal{N}_i^+ \\ 1 - \sum\limits_{j \in \mathcal{N}_i^+} q_{ij}, & j = i \\ 0, j \notin \mathcal{N}_i^+, & j \neq i \end{cases} \tag{12}$$

These matrices will be used in the following part to solve the optimization problem (6). Let $\mathcal{X}(k) = \{x_1(k), \cdots, x_{\mathcal{N}}(k)\}$ denote the state vector of all nodes in $\mathcal{G}$ at iteration k, $k \in \{1, 2, \cdots\}$. For the initial state $\mathcal{X}(0)$, the following two lemmas are given by referring to [36], which are very important for the algorithm design.

Lemma 1 (Consensus): If $\mathcal{G}$ is a strongly connected graph, for the discrete consensus algorithm $\mathcal{X}(k+1) = \mathcal{W}\mathcal{X}(k)$, there holds that $\lim_{t\to\infty} x_i(t) = c, \forall i \in \mathcal{V}$, where c is a constant.

Lemma 2 (Ratio Consensus): If $\mathcal{G}$ is a strongly connected graph, for the discrete ratio consensus algorithm $\mathcal{X}(k+1) = \mathcal{Q}\mathcal{X}(k)$, there holds that $\lim_{t\to\infty} x_i(t) = \beta_i \sum_{i=1}^{N} x_i(0), \forall i \in \mathcal{V}$, where β_i is the i-th element of the unit eigenvector corresponding to eigenvalue 1 of the matrix $\mathcal{Q}$.

Here, we point out that under Lemma 2, each node will converge to a stable state and the sum of all stable states will not change during the iteration process.

Algorithm 1 Consensus-Based EMA (CEMA)

Initialization:

Set $\lambda_i(0)$, $\mathcal{P}_i(0)$ and $\xi_i(0)$ $i, \forall i \in \mathcal{V}$ as below

$$\lambda_i(0) = \begin{cases} \dfrac{\mathcal{C}_i'(\mathcal{P}_i^m)}{(1 - 2\mathcal{B}_i\mathcal{P}_i^m)}, & i \in \mathcal{V}_g, \\ \mathcal{U}_i'(\mathcal{P}_i^M), & i \in \mathcal{V}_d, \end{cases} \tag{13}$$

$$\mathcal{P}_i(0) = 0, i \in \mathcal{V}, \tag{14}$$

$$\xi_i(0) = 0, i \in \mathcal{V}. \tag{15}$$

Given the small η satisfying $0 < \eta < 1$, and the termination errors ϵ_m and ϵ_l for the power mismatch and price, respectively.

Iteration: loop

1. Update λ_i according to
$$\lambda_i(k+1) = \sum_{j \in \mathcal{V}} w_{ij}\lambda_j(k) + \eta\xi_i(k), i \in \mathcal{V}. \tag{16}$$
2. Update $\mathcal{P}_i$ according to, when $i \in \mathcal{V}_g$,
$$\mathcal{P}_i(k+1) = \underset{\mathcal{P}_i^m \leq \mathcal{P}_i(k) \leq \mathcal{P}_i^M}{\arg\min} [\mathcal{C}_i(\mathcal{P}_i(k)) - \lambda_i(k+1)\mathcal{P}_i(k)]; \tag{17}$$
when $i \in \mathcal{V}_d$,
$$\mathcal{P}_i(k+1) = \underset{\mathcal{P}_i^m \leq \mathcal{P}_i(k) \leq \mathcal{P}_i^M}{\arg\min} [\lambda_i(k+1)\mathcal{P}_i(k) - \mathcal{U}_i(\mathcal{P}_i(k))]. \tag{18}$$
3. Update ξ_i according to, when $i \in \mathcal{V}_g$,
$$\xi_i(k+1) = \sum_{j \in \mathcal{V}} q_{ij}\xi_j(k) + \left(\mathcal{P}_i(k) - \mathcal{B}_i\mathcal{P}_i^2(k)\right) - \left(\mathcal{P}_i(k+1) - \mathcal{B}_i\mathcal{P}_i^2(k+1)\right); \tag{19}$$
when $i \in \mathcal{V}_d$,
$$\xi_i(k+1) = \sum_{j \in \mathcal{V}} q_{ij}\xi_j(k) + \mathcal{P}_i(k+1) - \mathcal{P}_i(k). \tag{20}$$
4. If $|\xi_i(k)| \leq \epsilon_m, \forall i \in \mathcal{V}$ and $|\lambda_i(k) - \lambda_i(k-1)| \leq \epsilon_l, \forall i \in \mathcal{V}$, break.

Output: $\lambda_i(k), P_i(k), \forall i \in \mathcal{V}$.

C. CEMA

In order to improve scalability and robustness, and reduce the operation information, a consensus-based distributed algorithm is proposed to solve problem (6). In the algorithm, each participant only uses local information (its own and neighbor incremental cost/utility and neighbor power mismatch estimation) to update its own state (its own incremental cost/utility, power and neighbor local power mismatch). The update rules are based on consensus [9], which guarantee the final value converges to the global optimal solution of problem (4).

Let $\xi_i(k), \forall i \in \mathcal{V}$ denote the local power mismatch of i at iteration k. The details can be found in CEMA shown in Algorithm 1.

For CEMA, in **Initialization**, each generator $i, i \in \mathcal{V}_g$ sets the initial price $\lambda_i(0)$ as the incremental price for the minimal generation power $\mathcal{P}_i^m$ to minimize its generation cost. To maximize its utility, each demand i sets the initial price as the incremental utility for the maximum demand power $\mathcal{P}_i^M$. Each local power mismatch $\xi_i(0), i \in \mathcal{V}$ is set to zero to guarantee that the total power mismatch is equal to the sum of the local power mismatch at initialization. In **Iteration**, update rule (16) promotes the convergence of both of price and the local mismatch. Based on the updated price, each generator (demand) i updates the generation (demand) power according to (17) ((18)) and then updates the local power mismatch according to (19) ((20)). The generator or demand power $\mathcal{P}_i, i \in \mathcal{V}$ is updated to maximize the profit of participant i The difference between (19) and (20) and the ratio consensus can guarantee the total power mismatch is equal to the sum of the local power mismatch for each iteration for directed connected communication network.

V. Analysis of CEMA

In this section, we provide the theoretical results regarding the convergence and the optimality of CEMA. Inspired by [20] and [26], we prove the convergence of CEMA firstly. Then, the optimality of CEMA is given under KKT conditions, which means that the final convergence value of CEMA is the optimal solution of the original problem (4).

A. Convergence of CEMA

Theorem 2: If generation capacity and demand capacity satisfy

$$\sum_{i\in\mathcal{V}_g} \mathcal{P}_i^m - \left(\mathcal{P}_i^m\right)^2 \le \sum_{i\in\mathcal{V}_d} \mathcal{P}_i^m \le \sum_{i\in\mathcal{V}_d} \mathcal{P}_i^M \le \sum_{i\in\mathcal{V}_g} \mathcal{P}_i^M - \left(\mathcal{P}_i^M\right)^2, \tag{21}$$

under CEMA, there exists $0 < \epsilon < 1$, such that when $0 < \eta < \epsilon$, there hold

$$\lim_{k\to\infty} \left[\sum_{i\in\mathcal{V}_d} \mathcal{P}_i(k) - \sum_{i\in\mathcal{V}_g} \left(\mathcal{P}_i(k) - \mathcal{B}_i \mathcal{P}_i^2(k) \right) \right] = 0, \tag{22}$$

$$\lim_{k\to\infty} \lambda_i(k) = \lambda_c, \quad \forall i \in \mathcal{V}, \tag{23}$$

and

$$\lim_{k\to\infty} \mathcal{P}(k) = \hat{\mathcal{P}}^*, \tag{24}$$

where λ_c is a constant and it satisfies (9), and $\hat{\mathcal{P}}^*$ is the convergence final vector of power.

The complete proof can be divided into three parts, i.e., global convergence, approximation and matrix disturbance, as shown in the Appendix B. In global convergence part, we obtain the dynamic of the global power mismatch and the incremental cost/utility, which shows that the global power mismatch will decrease and the incremental cost/utility will converge with iterations. Under these results, when the global power mismatch reaches a domain of small scale of asymptotic convergence, we investigate the behavior of the system by linearization. Therefore, we can get the matrix expression of the system and use the matrix disturbance theory to analyze the convergence. Finally, the convergence of CEMA is proved.

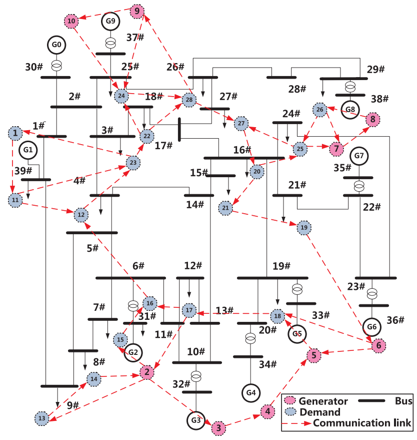


Fig. 1. IEEE 39-BUS System.

TABLE I
Parameters of Demands for IEEE 39-Bus

Demand Parameter				
Node	ω	α	$\mathcal{P}^m$	$\mathcal{P}^M$
11	18.43	0.0545	50	100.34
12	13.17	0.0877	100	159.13
13	15.46	0.0547	40	80.56
14	10.03	0.1041	30	123.98
15	8.45	0.0870	80	109.55
16	15.38	0.0984	40	76.34
17	19.16	0.1564	80	137.93
18	16.85	0.0564	50	84.19
19	15.63	0.0950	50	104.06
20	6.75	0.0470	78	119.36
21	14.95	0.0970	103	176.19
22	5.87	0.0349	33	88.65
23	18.18	0.0879	99	175.31
24	15.08	0.0653	89	129.03
25	14.90	0.0897	123	167.42
26	19.45	0.1345	5	36.51
27	18.05	0.0924	43	79.38
28	15.83	0.1026	67	147.26

TABLE II
Parameters of Generators for IEEE 39-Bus

Generator Parameter						
Node	c	b	a	$\mathcal{P}^m$	$\mathcal{P}^M$	$\mathcal{B}$
1	30	5.56	0.0024	60	339.69	0.00021
2	25	4.32	0.0056	25	479.10	0.00031
3	25	6.60	0.0072	28	290.4	0.00011
4	16	3.14	0.0047	40	306.34	0.00022
5	6	7.54	0.0091	35	593.80	0.00041
6	54	3.28	0.0018	29	137.19	0.00051
7	23	7.31	0.0053	45	595.40	0.000451
8	15	2.45	0.0063	56	162.17	0.000351
9	20	7.63	0.0028	12	165.1	0.000331
10	12	4.76	0.0046	30	443.41	0.000121

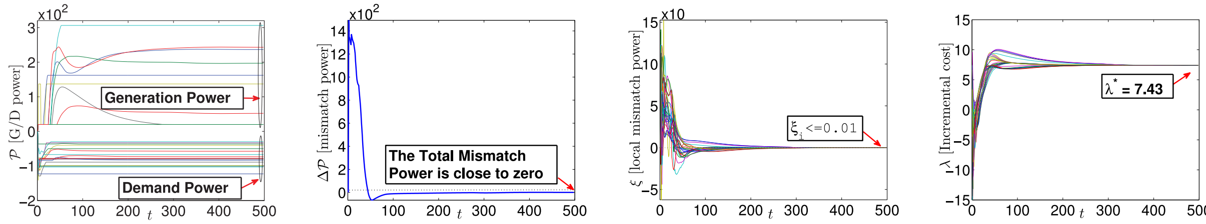
B. Optimality of CEMA

The convergence of CEMA has been proved in the above subsection. Here, we prove the optimality of CEMA, which means that CEMA converges to the optimal solution of problem (4).

Theorem 3: If (7) holds, under CEMA, we have

$$\lim_{k\to\infty} \mathcal{P}(k) = \hat{\mathcal{P}}^* = \mathcal{P}^*,$$

where $\mathcal{P}^*$ is the optimal solution of problem (4).



(a) G/D Power (b) Mismatch Power (c) Parameter ξ (d) Electricity Price

Fig. 2. Convergence of $\mathcal{P}$, $\triangle\mathcal{P}$, ξ and λ under CEMA.

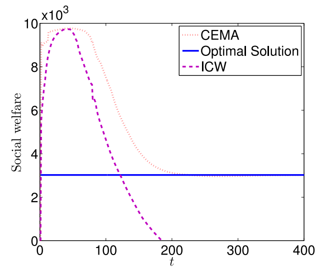
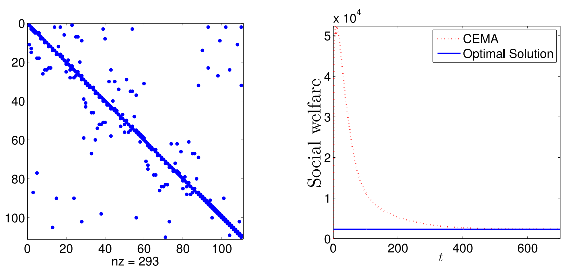


Fig. 3. Optimality of CEMA.



(a) Structure of $\mathcal{W}$ (b) CEMA Performance

Fig. 4. IEEE 118-bus system.

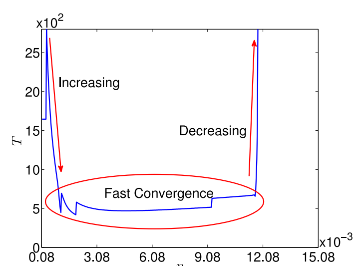
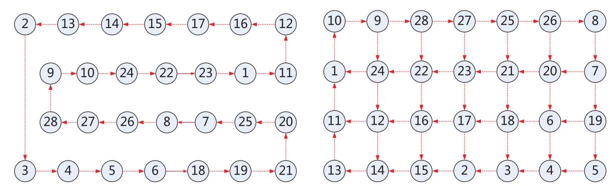


Fig. 5. CEMA Performance VS η.

To prove Theorem 3, we firstly prove that the final value of $\mathcal{P}(k)$ can always make KKT conditions of the convex problem (6) hold. Since problem (4) has the same optimal solution as problem (6) under condition (7), the final value of $\mathcal{P}(k)$ is the optimal solution of problem (4). The detailed proof can be seen in the Appendix C.



(a) Ring (b) Grid

Fig. 6. Different Directed topology.

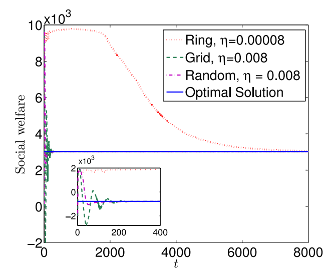
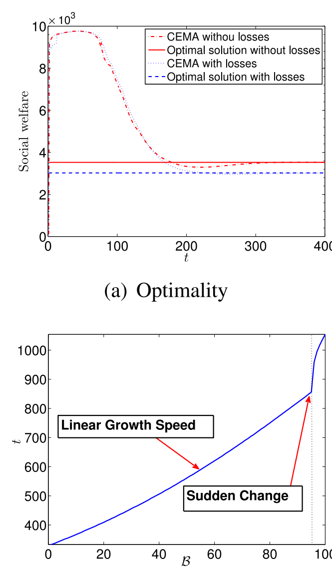


Fig. 7. CEMA Performance under Different Directed Topologies.



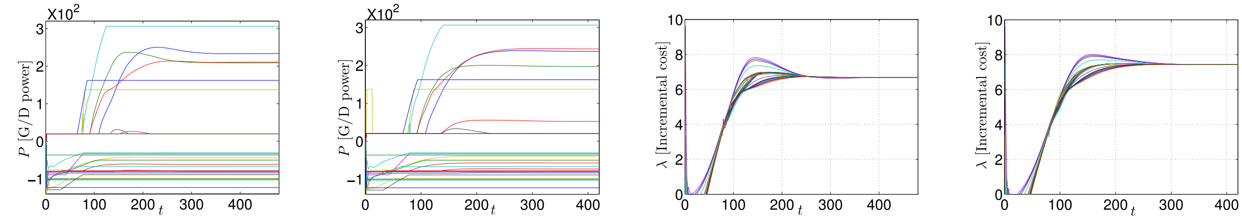
(a) Optimality

(b) Convergence

Fig. 8. Optimality and Convergence VS Parameter $\mathcal{B}$.

VI. Performance Evaluation

In this section, extensive simulations are conducted to illustrate our results in terms of convergence, optimality for different directed communication topologies, and we also illustrate the effect of the tuning parameter η and transmission losses.



(a) G/D Power without Losses (b) G/D Power with Losses (c) Electricity Price without Losses (d) Electricity Price with Losses

Fig. 9. Comparison of Considering and Not Considering Losses.

A. Parameter Setting

We consider the IEEE 39-bus system with 10 generation units and 18 demands. The communication network of these generation units and demands, which is described by the red lines in Fig. 1, is directed connected. The utility function of each demand $j, \forall j \in \mathcal{V}_d$ is chosen as (1). The parameters of the demands and the generators are provided in Tab. I and Tab. II by referring to [26]. Here, we assume the lower bound for demand power is nonzero and the lower bound for generator unit is nonzero. We set $\eta = 0.00786$, $\epsilon_m = 0.01$ and $\epsilon_l = 0.01$.

B. Performance of CEMA

1) Convergence and Optimality: In order to illustrate the convergence and optimality of CEMA, we conduct our algorithm under the above scenario and parameters setting. The verification of the CEMA convergence performance is shown in Fig. 2. The generators power and the demands power distribution are shown in Fig. 2(a), where each demand power $\mathcal{P}_i, i \in V_d$ is shown as the negative one to distinguish from the generator units. As shown in Fig. 2(b), the total power mismatch will converge to zero with iterations, while all local mismatch power $\xi_i, \forall i \in \mathcal{V}$ get close to zero shown in Fig. 2(c). It can be observed that the equality constraint (4a) will be satisfied gradually. Due the existence of mismatch power variance caused by the initialization (15), the parameter $\xi_i, \forall i \in \mathcal{V}$ becomes nonzero from beginning, and then gets back to zero gradually. The optimal price $\lambda^* = 7, 43$ is achieved by both generators and demands in a distributed way, which can be observed from Fig. 2(a). Under the final price, all participants maximize their profit. When $t = 497$, CEMA converges to the result which satisfies the termination conditions $\epsilon_m = 0.01$ and $\epsilon_l = 0.01$. We also compare our result with the optimal solution using global information and the ICW algorithm [26] in Fig. 3. While ICW algorithm diverges with iteration k for the directed connected communication network, CEMA algorithm achieves optimal solution. The total welfare under CEMA is 3021.7, which is very close to the optimal one 3021.6. The square distance between the solution $\mathcal{P}^*$ under CEMA and the optimal one is about 0.0293, which is negligible.

To illustrate the scalability of CEMA, we evaluate its convergence and optimality performance on IEEE 118-bus system. The system is composed of 19 generator units and 91 loads. The structure of the weight matrix $\mathcal{W}$ is shown in Fig. 4(a), which also reflects the topology of the communication network. It can be observed in Fig. 4(b) that under CEMA, social welfare maximization can be achieved with an exponential speed.

2) Performance Under Different η: To discuss the performance of CEMA with various tuning parameter η, we conduct the simulation in IEEE-39 bus system as shown in Fig. 1. The initial η is set to be $\eta = 1.1 \times 10^{-4}$ and the increment for each time test is set to be $\triangle\eta = 3 \times 10^{-4}$. It is observed from Fig. 5 that with the growth of η, the iteration T for convergence decreases firstly and then increases. When η is too large, CEMA cannot converge and there exists an optimal η for fast convergence.

3) Performance Under Different Directed Topologies: Simulations are conducted to illustrate the performance of CEMA under different directed communication topologies, Specifically, three topologies are considered, i.e., randomly generated topology in Fig. 1, ring topology in Fig. 6(a) and grid topology (7×4) shown in Fig. 6(b). It can be observed from Fig. 7 that under the ring topology, CEMA needs more than 8000 iterations to converge and requires a very small parameter $\eta = 0.00008$ compared with the random and grid network where $\eta = 0.008$ and they only need 400 iterations. It means that under topology with better connectivity, CEMA can converge with a faster speed and the upper bound of η becomes larger.

4) Losses Effect: Here, we investigate how the parameter $\mathcal{B}$ affects the total welfare in the above scenario firstly and then how the transmission losses affect the convergence speed of CEMA in IEEE-9 bus system with 3 generator buses and 3 demands. It can be observed in Fig. 8(a) that the existence of transmission losses will decrease the total welfare compared with that of case without losses. We observe that considering transmission losses, the dispatch of generation and the response of demand change shown in Fig. 9(a) compared with Fig. 9(b). The electricity price rises shown in Fig. 9(c) compared with Fig. 9(d), which is caused by power losses.

Consider that the initial losses coefficient is $\mathcal{B} = [5, 7, 4]' \times 10^{-4}$ and the increment for each step shown in the horizontal axis is $\triangle\mathcal{B} = [10, 12, 7]' \times 10^{-6}$ for the IEEE-9 bus system. The relationship between the convergence speed and the losses parameter is shown in Fig. 8(b). The overall tendency can be observed that with the growth of $\mathcal{B}_i, \forall i \in \mathcal{V}$, the convergence iteration will become large. When $\mathcal{B}_i, \forall i \in \mathcal{V}$ is too large, CEMA will not converge because with large $\mathcal{B}_i, \forall i \in \mathcal{V}$, (21) is no longer valid.

VII. Conclusion

By taking transmission losses and the directed information flow into consideration, we provided a consensus-based distributed energy management (CEMA) strategy for distributed generation units and responsive demands. Under CEMA, the global price and the global power balance between generation and demand can be achieved under consensus iteration. Especially, ratio consensus was utilized to guarantee that the global power balance can be realized with local information exchange and iteration. We prove that when the total maximum demand power is larger than or equal to the total minimal generation one subtracted the transmission losses, CEMA can achieve the social welfare maximization solution asymptotically when the optimization problem is non-convex. In simulation, we observed that considering transmission losses, social welfare decreased and the convergence speed reduced with the growth of transmission losses. Fast convergence with lower cost and lower complexity, more practical scenario modeling and analysis will be our future work.

Appendix A
Proof of Theorem 1

Proof 1: Since the converted problem (problem (6) in Section III-A) is a convex optimization problem, the optimal value of the Lagrange dual problem has strong duality. The Lagrangian function of the converted problem is

$$\begin{aligned}\tilde{L}(\mathcal{P},\lambda,\gamma,\vartheta) &= \sum_{j\in\mathcal{V}_d} -\mathcal{U}_j(\mathcal{P}_j) + \sum_{i\in\mathcal{V}_g} \mathcal{C}_i(\mathcal{P}_i)\\ &\quad + \lambda\left(\sum_{j\in\mathcal{V}_d}\mathcal{P}_j + \sum_{i\in\mathcal{V}_g}\left(\mathcal{B}_i(\mathcal{P}_i)^2 - \mathcal{P}_i\right)\right)\\ &\quad + \sum_{i\in\mathcal{V}}\gamma_i\left(\mathcal{P}_i^m - \mathcal{P}_i\right) + \sum_{i\in\mathcal{V}}\vartheta_i\left(\mathcal{P}_i - \mathcal{P}_i^M\right),\end{aligned} \tag{A1}$$

where $\gamma, \vartheta \in R^N$, $\lambda \in R$, $\gamma_i \geq 0$, $\vartheta \geq 0$, $\lambda \geq 0$, $\forall i \in \mathcal{V}$.

According to KKT conditions, we have

$$\frac{\partial \tilde{L}}{\partial \mathcal{P}_j^*} = \lambda - \gamma_j + \vartheta_j - \frac{d\mathcal{U}_j\left(\mathcal{P}_j^*\right)}{d\mathcal{P}_j^*} = 0, \quad \forall j \in \mathcal{V}_d \tag{A2a}$$

$$\frac{\partial \tilde{L}}{\partial \mathcal{P}_i^*} = \frac{d\mathcal{C}_i\left(\mathcal{P}_i^*\right)}{d\mathcal{P}_i^*} - \lambda\left(1 - 2\mathcal{B}_i\mathcal{P}_i^*\right) - \gamma_i + \vartheta_i = 0, \quad \forall i \in \mathcal{V}_g \tag{A2b}$$

$$\lambda\left(\sum_{i\in\mathcal{V}_d}\mathcal{P}_i^* + \sum_{i\in\mathcal{V}_g}\left(\mathcal{B}_i\left(\mathcal{P}_i^*\right)^2 - \mathcal{P}_i^*\right)\right) = 0, \lambda \geq 0 \tag{A2c}$$

$$\gamma_i\left(\mathcal{P}_i^m - \mathcal{P}_i^*\right) = 0, \gamma_i \geq 0, \quad \forall i \in \mathcal{V} \tag{A2d}$$

$$\vartheta_i\left(\mathcal{P}_i^* - \mathcal{P}_i^M\right) = 0, \vartheta_i \geq 0, \quad \forall i \in \mathcal{V} \tag{A2e}$$

Next, we prove our result by contradiction. Suppose that

$$\sum_{j\in\mathcal{V}_d}\mathcal{P}_j^* < \sum_{i\in V_g}\left(\mathcal{P}_i^* - \mathcal{B}_i\left(\mathcal{P}_i^*\right)^2\right). \tag{A3}$$

Thus, for the condition (A2c), it holds $\lambda = 0$.

Consider the general case $\mathcal{P}_i^m \neq \mathcal{P}_i^M, \forall i \in \mathcal{V}$, because $\mathcal{P}_i^m \leq \mathcal{P}_i^* \leq \mathcal{P}_i^M$, at least one of γ_i and $\vartheta_i, \forall i \in \mathcal{V}$ is equal to 0 to guarantee that conditions (A2d) and (A2e) hold. According to the property of the utility function, consider the general case, $\frac{d\mathcal{U}_j(\mathcal{P}_j^*)}{d\mathcal{P}_j^*} > 0, \forall j \in \mathcal{V}_d$. Hence $\gamma_j = 0, \vartheta_j > 0$, $\forall j \in \mathcal{V}_d$. There hold

$$\frac{\partial \tilde{L}}{\partial \mathcal{P}_j^*} = -\gamma_j + \vartheta_j - \frac{d\mathcal{U}_j(\mathcal{P}_j^*)}{d\mathcal{P}_j^*} = 0, \forall j \in \mathcal{V}_d$$

and

$$\vartheta_j\left(\mathcal{P}_j^* - \mathcal{P}_j^M\right) = 0, \vartheta_j \geq 0, \forall j \in \mathcal{V}_d.$$

Thus, we have $\mathcal{P}_j^* = \mathcal{P}_j^M$, $\forall j \in \mathcal{V}_d$. For the same reason, $\mathcal{P}_i^* = \mathcal{P}_i^m$, $\forall i \in \mathcal{V}_g$. It further follows from (A3) that

$$\sum_{j\in\mathcal{V}_d}\mathcal{P}_j^M < \sum_{i\in\mathcal{V}_g}\left(\mathcal{B}_i\left(\mathcal{P}_i^m\right)^2 - \mathcal{P}_i^m\right). \tag{A4}$$

Note that (A4) is contradicted with the sufficient condition (inequality (7) referred in Theorem 1). Hence, (A3) cannot hold for $\mathcal{P}^*$ under the sufficient condition. Because of the first constraint in the converted problem, for the optimal solution $\mathcal{P}^*$ of the converted problem, there must hold

$$\sum_{j\in\mathcal{V}_d}\mathcal{P}_j^* = \sum_{i\in\mathcal{V}_g}\left(\mathcal{P}_i^* - \mathcal{B}_i\left(\mathcal{P}_i^*\right)^2\right).$$

Therefore, the converted problem always achieves the optimal solution with the first constraint obtaining the equal sign under the sufficient condition. The feasible set of the converted problem contains the feasible set of the original problem (problem (4) in Section II-C) and their objective functions are the same. Hence, the optimal solution of the converted problem is less than or equal to that of the original problem. Since the optimal solution of the converted problem is always achieved in the feasible set of the original problem under the sufficient condition, the optimal solution of the converted problem is same as that of the original problem.

Appendix B
Proof of Theorem 2

Proof 2 (Global Convergence): Define the mismatch power between generation and demand as

$$\Delta\mathcal{P}(k) = \sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k) - \sum_{i\in\mathcal{V}_g}\left(\mathcal{P}_i(k) - \mathcal{B}_i\mathcal{P}_i^2(k)\right).$$

Summing all $\xi_i(k+1), \forall i \in \mathcal{V}$, it follows from step 4 in CEMA and Lemma 2 that

$$\begin{aligned}\sum_{i\in\mathcal{V}}\xi_i(k+1) &= \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} q_{ij}\xi_j(k) + \sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k+1) - \sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k)\\ &\quad + \sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i(k) - \mathcal{B}_i\mathcal{P}_i^2(k)\Big)\\ &\quad - \sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i(k+1) - \mathcal{B}_i\mathcal{P}_i^2(k+1)\Big)\\ &= \sum_{i\in\mathcal{V}}\xi_i(k) + \left(\sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i(k+1) - \mathcal{B}_i\mathcal{P}_i^2(k+1)\Big) - \sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k+1)\right)\\ &\quad - \left(\sum_{i\in\mathcal{V}_g}\Big(\mathcal{P}_i(k) - \mathcal{B}_i\mathcal{P}_i^2(k)\Big) - \sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k)\right)\\ &= \sum_{i\in\mathcal{V}}\xi_i(k) + \Delta\mathcal{P}(k+1) - \Delta\mathcal{P}(k). \qquad \text{(A5)}\end{aligned}$$

Then, from initialization of $\mathcal{P}_i(0), \xi_i(0), \forall i \in \mathcal{V}$ under CEMA, we have

$$\begin{aligned}\sum_{i\in\mathcal{V}}\xi_i(k+1) - \Delta\mathcal{P}(k+1) &= \sum_{i\in\mathcal{V}}\xi_i(k) - \Delta\mathcal{P}(k)\\ &= \sum_{i\in\mathcal{V}}\xi_i(0) - \Delta\mathcal{P}(0)\\ &= 0, \qquad \text{(A6)}\end{aligned}$$

i.e., $\sum_{i\in\mathcal{V}}\xi_i(k) = \Delta\mathcal{P}(k)$. Summing all $\lambda_i(k+1), \forall i \in \mathcal{V}$, one infers

$$\sum_{i\in\mathcal{V}}\lambda_i(k+1) = \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} w_{ij}\lambda_j(k) + \eta\sum_{i\in\mathcal{V}}\xi_i(k), \qquad \text{(A7)}$$

where η can be seen as a proportional feedback gain. According to step 2 in CEMA, all $\lambda_i(k+1), i \in \mathcal{V}$ will be close to each other for $k = 0$ with zero feedback. And then we get $\mathcal{P}_i(k), i \in \mathcal{V}$ according to step 3 and 4, where $k = 1$.

1) Consider $\Delta\mathcal{P}(k) > 0$. From (A7), when $\sum_{i\in\mathcal{V}}\xi_i(k) > 0$, $\sum_{i\in\mathcal{V}}\lambda_i(k+1)$ will increase and all $\lambda_i(k+1), i \in \mathcal{V}$ approach to the same value according to Lemma 1. Note that before reaching the upper or lower bound, we always have $\mathcal{P}_i(k) = \frac{\lambda_i(k)-b_i}{2a_i+2\mathcal{B}_i\lambda_i(k)}, \forall i \in \mathcal{V}_g$. It means that the power generation is a monotonically increasing function of incremental cost $\lambda_i(k), \forall i \in \mathcal{V}_g$. Meanwhile, note that $\mathcal{P}_i(k) = \mathcal{U}'^{-1}(\lambda_i(k)), \forall i \in \mathcal{V}_d$. Since $\mathcal{U}' \le 0$, the power demand is a non-increasing function of $\lambda_i(k), \forall i \in \mathcal{V}_d$. Therefore, the total power generation $\sum_{i\in\mathcal{V}_g}\mathcal{P}_i(k)$ will increase and the total power demand $\sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k)$ will decrease or keep unchanged.

2) Consider $\Delta\mathcal{P}(k) < 0$. From (A7), when $\sum_{i\in\mathcal{V}}\xi_i(k) < 0$, $\sum_{i\in\mathcal{V}}\lambda_i(k+1)$ will decrease and all $\lambda_i(k+1), i \in \mathcal{V}$ approach to the same value according to Lemma 1. For the same reason in 1), the total power generation $\sum_{i\in\mathcal{V}_g}\mathcal{P}_i(k)$ will decrease and the total power demand $\sum_{i\in\mathcal{V}_d}\mathcal{P}_i(k)$ will increase or keep unchanged.

Thus, the method in (A7) will decrease the mismatch absolute value of $\Delta\mathcal{P}(k)$ and all $\lambda_i, i \in \mathcal{V}$ will approach to the same value.

After some sufficiently long time $\mathcal{K}$, if the mismatch $\Delta\mathcal{P}(\mathcal{K})$ reaches a domain of small-scale asymptotic convergence, then $\Delta\mathcal{P}(\mathcal{K})$ will keep increasing (or decreasing) for $k \ge \mathcal{K}$.

Approximation: To investigate the behavior for $k \ge \mathcal{K}$, we will linearize the update in each iteration. Seeing that the iteration for $\mathcal{P}_i(k)$ in step 3 of CEMA is a continuous-time function. Hence, it follows

$$\lambda_i(t) = \mathcal{U}_i'(\mathcal{P}_i(t)), t \in [k, k+1].$$

According to the Taylor Theorem, there hold

$$\begin{aligned}\lambda_i(t_0) + \Delta\lambda_i(t) &\approx \mathcal{U}_i'(\mathcal{P}_i(t_0)) + \mathcal{U}_i''(\mathcal{P}_i(t_0))\Delta\mathcal{P}_i(t),\\ \Delta\lambda_i(t) &\approx \mathcal{U}_i''(\mathcal{P}_i(t_0))\Delta\mathcal{P}_i(t).\end{aligned}$$

From iteration k to $k+1$, considering that $\min\{\mathcal{U}_i''(\mathcal{P}_i(t)) < 0, t \in [k, k+1]\}$, there exists $0 \le -\delta_i(k) \le -\frac{1}{\min\{\mathcal{U}_i''(\mathcal{P}_i(t)), t\in[k,k+1]\}}$, such that

$$\mathcal{P}_i(k+1) = \mathcal{P}_i(k) + \delta_i(k)(\lambda_i(k+1) - \lambda_i(k)). \qquad \text{(A8)}$$

Consider that $\min\{\mathcal{U}_i''(\mathcal{P}_i(t)) < 0, t \in [k, k+1]\}$, we have that there exists $-\delta_i(k) = 0$, such that (A8) holds.

Combining step 4 in CEMA and the above result, it yields that

$$\xi_i(k+1) = \sum_{j\in\mathcal{V}q_{ij}}\xi_j(k) + \delta_i(k)(\lambda_i(k+1) - \lambda_i(k)), i \in \mathcal{V}_d.$$

Then for the generator i which does not reach its upper bound or lower bound, there holds

$$\begin{aligned}&\mathcal{P}_i(k) - \mathcal{P}_i(k+1)\\ &= \frac{\lambda_i(k) - b_i}{2a_i + 2\mathcal{B}_i\lambda_i(k)} - \frac{\lambda_i(k+1) - b_i}{2a_i + 2\mathcal{B}_i\lambda_i(k+1)}\\ &= \frac{(\lambda_i(k) - b_i)(2a_i + 2\mathcal{B}_i\lambda_i(k+1)) - (\lambda_i(k+1) - b_i)(2a_i + 2\mathcal{B}_i\lambda_i(k))}{(2a_i + 2\mathcal{B}_i\lambda_i(k))(2a_i + 2\mathcal{B}_i\lambda_i(k+1))}\\ &= \frac{(\lambda_i(k) - \lambda_i(k+1))(2a_i + 2\mathcal{B}_ib_i)}{(2a_i + 2\mathcal{B}_i\lambda_i(k))(2a_i + 2\mathcal{B}_i\lambda_i(k+1))}.\end{aligned}$$

There exists $\delta_i(k)$ which satisfies

$$0 \le -\delta_i(k) \le \max\left\{\frac{(2a_i + 2\mathcal{B}_ib_i)}{(2a_i + 2\mathcal{B}_i\lambda_i(k))(2a_i + 2\mathcal{B}_i\lambda_i(k))}, \frac{(2a_i + 2\mathcal{B}_ib_i)}{(2a_i + 2\mathcal{B}_i\lambda_i(k+1))(2a_i + 2\mathcal{B}_i\lambda_i(k+1))}\right\}$$

such that

$$\xi_i(k+1) = \sum_{j\in\mathcal{V}q_{ij}}\xi_j(k) + \delta_i(k)(\lambda_i(k+1) - \lambda_i(k)), i \in \mathcal{V}_g.$$

Therefore, the matrix form of iteration process after $\mathcal{K}$ can be written as

$$\begin{bmatrix}\lambda(k+1)\\ \xi(k+1)\end{bmatrix} = \begin{bmatrix}\mathcal{W} & \eta I\\ -\delta(k)(I - \mathcal{W}) & \mathcal{Q} + \eta\delta(k)\end{bmatrix}\begin{bmatrix}\lambda(k)\\ \xi(k)\end{bmatrix}. \qquad \text{(A9)}$$

Define $\Gamma(k) = \begin{bmatrix} \mathcal{W} & 0 \\ -\delta(k)(I-\mathcal{W}) & \mathcal{Q} \end{bmatrix}$ and $\Upsilon(k) = \begin{bmatrix} 0 & I \\ 0 & \delta(k) \end{bmatrix}$.

Matrix Disturbance: The system (A9) can be seen as a linear time varying system where the system matrix $\Gamma(k)$ at iteration k perturbed by $\eta\Upsilon(k)$. $\Gamma(k)$ is a lower block triangular matrix and all its eigenvalues are the union of the eigenvalues of $\mathcal{W}$ and $\mathcal{Q}$. So $\Gamma(k)$ has two eigenvalues $\sigma_1(k) = \sigma_2(k)$, and the rest eigenvalues lie in the open unit disk on the complex plane. Constructing vectors $\mu_1(k)$, $\mu_2(k)$ and $\nu_1(k)$, $\nu_2(k)$ as follows:

$$U = [\,\mu_1(k)\ \mu_2(k)\,] = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ u & \psi(k)u \end{bmatrix},$$

where $\psi(k) = \sum_{i\in\mathcal{V}} \delta_i(k)$ and u is the unit eigenvector corresponding to eigenvalue 1 of the matrix $\mathcal{W}$. And

$$V^T(k) = \begin{bmatrix} \nu_1^T(k) \\ \nu_2^T(k) \end{bmatrix} = \begin{bmatrix} -\mathbf{1}^T\delta(k) & \mathbf{1}^T \\ \omega^T & \mathbf{0}^T \end{bmatrix},$$

where $\mathbf{1}$ is a vector of length N with all its elements being 1, $\delta(k) = diag\{\delta_1(k), \cdots, \delta_N(k)\}$ and ω is the unit eigenvector corresponding to eigenvalue 1 of the matrix $\mathcal{Q}$. Hence, $V^T(k)U(k) = I$.

When η is small, the variation of $\sigma_1(k)$ and $\sigma_2(k)$ perturbed by $\eta\Upsilon(k)$ can be quantified by the eigenvalues of $V^T(k)\Upsilon(k)U(k)$, and

$$V^T(k)\Upsilon(k)U(k) = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \omega^T u & \psi(k)\omega^T u \end{bmatrix}.$$

The eigenvalues of $V^T(k)\Upsilon(k)U(k)$ are 0 and $\psi(k)\omega^T u < 0$. Thus, $d\sigma_1(k)/d\eta = 0$ and $d\sigma_2(k)/d\eta = \psi(k)\omega^T u < 0$. It means that $\sigma_1(k)$ does not change against η, and when $\eta > 0$, $\sigma_2(k)$ becomes smaller. Let ζ be the upper bound of η. If $0 < \eta < \zeta$, we have $|\sigma_2(k)| < 1$. Since eigenvalues continuously depend on the entries of a matrix. In this case, the rest eigenvalues of $\Gamma(k) + \eta\Upsilon(k)$ continuously depend on η. For the same reason, there exists an upper bound ζ' such that if $0 < \eta < \zeta'$, we have $|\sigma_i(k)| < 1, i = 3, 4, \ldots 2N$. Hence, if $\eta < \epsilon = \min\{\zeta, \zeta'\}$, $\sigma_1(k) = 1$ is simple and all rest eigenvalues are within the open unit disk.

By verification, $\begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}$ is the eigenvector of the system matrix associated with $\sigma_1(k) = 1$. Because all the rest eigenvalues are within the open unit disk, it follows

$$\lim_{k\to\infty} \begin{bmatrix} \lambda(k) \\ \xi(k) \end{bmatrix} \to span \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}.$$

Therefore, $\lim_{k\to\infty} \xi_i(k) = 0$ and the power generation is equal to the power demand, which means that

$$\lim_{k\to\infty} \left[\sum_{i\in\mathcal{V}_d} \mathcal{P}_i(k) - \sum_{i\in\mathcal{V}_g} \left(\mathcal{P}_i(k) - \mathcal{B}_i \mathcal{P}_i^2(k) \right) \right] = 0.$$

And $\lim_{k\to\infty} \lambda_i(k) = \lambda_c$ converges to a common value.

From the above analysis, the global convergence shows that all $\lambda_i(k), \forall i \in \mathcal{V}$ will become closer to each other, while the mismatch power will become smaller. Approximation analysis expresses that the iteration process can be approximated as a matrix form, and the matrix disturbance analysis shows $\lambda_i(k), \forall i \in \mathcal{V}$ will achieve consensus and the mismatch power will become zero. Hence, one can conclude that the results in Theorem 2 (equalities (22), (23) and (24) referred in Theorem 2) will be achieved under CEMA.

Appendix C
Proof of Theorem 3

Proof 3: According to Theorem 1, one infers that if the sufficient condition holds, the optimal solution of the converted problem equals to that of the original problem. Hence, $\mathcal{P}^*$ is also the optimal solution of the converted problem and we just need to prove that the convergence results shown in (22), (23) and (24) can always satisfy the KKT conditions (A2) of the convex converted problem.

From (22), we have $\sum_{i\in\mathcal{V}_d} \hat{\mathcal{P}}_i^* = \sum_{i\in V_g} (\hat{\mathcal{P}}_i^* - \mathcal{B}_i(\hat{\mathcal{P}}_i^*)^2)$. It means that the power supplied by generators and the power consumed by the demands are balanced under CEMA and the KKT condition (A2c) holds.

From step 2, 3, 4 in CEMA, there holds

$$\lim_{k\to\infty} \mathcal{P}_i(k) = \underset{\mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M, i\in\mathcal{V}_d}{\arg\min} \lambda_c \lim_{k\to\infty} \mathcal{P}_i(k) - \mathcal{U}_i\left(\lim_{k\to\infty} \mathcal{P}_i(k)\right).$$

To analyze the above equation, for each demand i, consider the following equivalent optimization problem,

$$\begin{aligned} \min \quad & \lambda_c \mathcal{P}_i - \mathcal{U}_i(\mathcal{P}_i) \\ s.t \quad & \mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M, i \in \mathcal{V}_d. \end{aligned} \tag{A10}$$

It can be seen that the above optimization problem is a convex optimization problem. $\hat{\mathcal{P}}_i^*$. Considering that λ_c satisfies $\lambda_c = \frac{d\mathcal{U}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$ and $\hat{\mathcal{P}}_i^*$ satisfies $\mathcal{P}_i^m < \hat{\mathcal{P}}_i^* < \mathcal{P}_i^M$. According to the KKT conditions (A2), if $\mathcal{P}_i^m < \mathcal{P}_i^* < \mathcal{P}_i^M$ for some $i \in \mathcal{V}_d$, there hold $\gamma_i = 0$, $\vartheta_i = 0$ and $\lambda = \frac{d\mathcal{U}_i(\mathcal{P}_i^*)}{d\mathcal{P}_i^*}$. Hence, (A2a), (A2d) and (A2e) will be satisfied under CEMA for all $i \in \mathcal{V}_d$.

For the same reason, for $i \in \mathcal{V}_g$, considering that λ_c satisfies $\lambda_c = \frac{d\mathcal{C}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$ and we have $\mathcal{P}_i^m < \hat{\mathcal{P}}_i^* < \mathcal{P}_i^M$. Hence, under our algorithm, and there holds $\lim_{k\to\infty} \lambda_i(k) = \lambda_c = \frac{d\mathcal{C}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$, (A2b), (A2d) and (A2e) will be satisfied under CEMA for all $i \in \mathcal{V}_g$.

For the demand i, when $\lambda_c \ge \frac{d\mathcal{U}_i(\mathcal{P}_i^m)}{d\mathcal{P}_i^m}$ ($\lambda_c \le \frac{d\mathcal{U}_i(\mathcal{P}_i^M)}{d\mathcal{P}_i^M}$), without the boundary constraint $\mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M$, the optimal solution satisfies $\lambda_c = \frac{d\mathcal{U}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$. When $\mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M$, $\frac{d\mathcal{U}_i(\mathcal{P}_i)}{d\mathcal{P}_i}$ is a decreasing function. Hence, when $\mathcal{P}_i^m \le \mathcal{P}_i \le \mathcal{P}_i^M$, the differential of the objective function in problem (A10) is a increasing function (decreasing function) of $\mathcal{P}_i$. Therefore, the optimization problem (A10) will reach its minimum value and the optimal solution satisfies $\hat{\mathcal{P}}_i^* = \mathcal{P}_i^m$ ($\hat{\mathcal{P}}_i^* = \mathcal{P}_i^M$).

When $\hat{\mathcal{P}}_i^* = \mathcal{P}_i^m, i \in \mathcal{V}_d$ ($\hat{\mathcal{P}}_i^* = \mathcal{P}_i^M, i \in \mathcal{V}_d$), there hold $\gamma_i \geq 0$, $\vartheta_i = 0$ and $\lambda_c - \gamma_i = \frac{d\mathcal{U}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$ ($\gamma_i = 0$, $\vartheta_i \geq 0$ and $\lambda_c - \vartheta_i = \frac{d\mathcal{U}_i(\hat{\mathcal{P}}_i^*)}{d\hat{\mathcal{P}}_i^*}$). Hence, (A2a), (A2d) and (A2e) will be satisfied under CEMA for all $i \in \mathcal{V}_d$.

For the same reason, when $i \in \mathcal{V}_g$ and $\mathcal{P}_i^m = \hat{\mathcal{P}}_i^*$ ($\hat{\mathcal{P}}_i^* = \mathcal{P}_i^M$, under our algorithm, there holds $\lim_{k\to\infty} \lambda_i(k) \leq \frac{d\mathcal{C}_i(\mathcal{P}_i^m)}{d\mathcal{P}_i^m}$ ($\lim_{k\to\infty} \lambda_i(k) \geq \frac{d\mathcal{C}_i(\mathcal{P}_i^m)}{d\mathcal{P}_i^m}$). Hence, (A2b), (A2d) and (A2e) will be satisfied under CEMA for all $i \in \mathcal{V}_g$.

Hence, under our algorithm, the KKT condition can always be tenable. This means that the final value of our algorithm is the optimal solution of both the converted problem and the original one.

Acknowledgment

The authors would like to thank the editors and the anonymous reviewers for their helpful and constructive comments, which have helped us improve the manuscript significantly.

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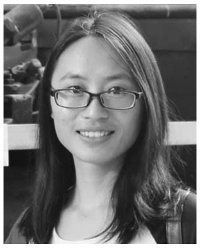

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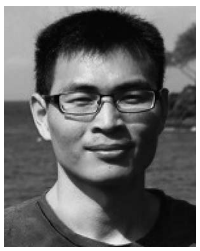

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